電弱精密測定と標準模型を超える物理

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§.0 電弱精密測定

	uncertainty
$\alpha^{-1}(M_Z) = 128.91(2)$	1.6×10^{-4}
$M_Z = 91.1876(21)\text{GeV}$	2.3×10^{-5}
$G_F = 1.16637(1) \times 10^{-5}\text{GeV}^{-2}$	9×10^{-6}

Erler-Langacker in RPP2006

We know three input parameters of the standard model ($SU(2)$ and $U(1)$ gauge couplings, g_W, g_Y, and the VEV of Higgs, v) within 10^{-4} accuracy.

Many values are precisely measured within 10^{-3} accurary

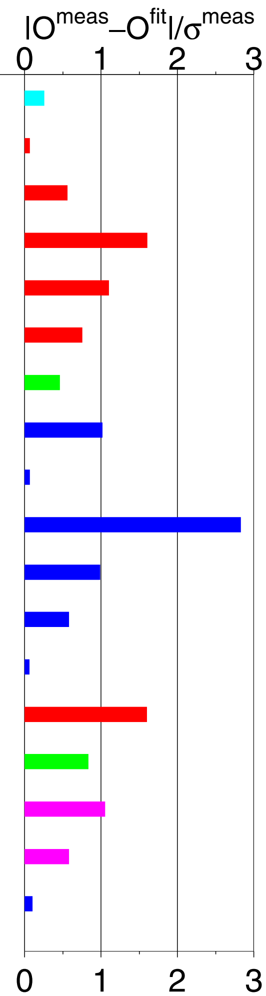
	uncertainty
$\Gamma_Z = 2.4952(23)\text{GeV}$	9.2×10^{-4}
$\sin^2\theta_{\text{eff}}^{\text{lept}} = 0.23153(16)$	6.9×10^{-4}
$M_W = 80.403(29)\text{GeV}$	3.6×10^{-4}

http://lepewwg.web.cern.ch/LEPEWWG/
RPP2006

at Z and W poles.

標準模型を 10^{-3} の精度でテストできる!!

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^{0} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



あらゆる測定結果が標準模型の予言にぴったり一致している!

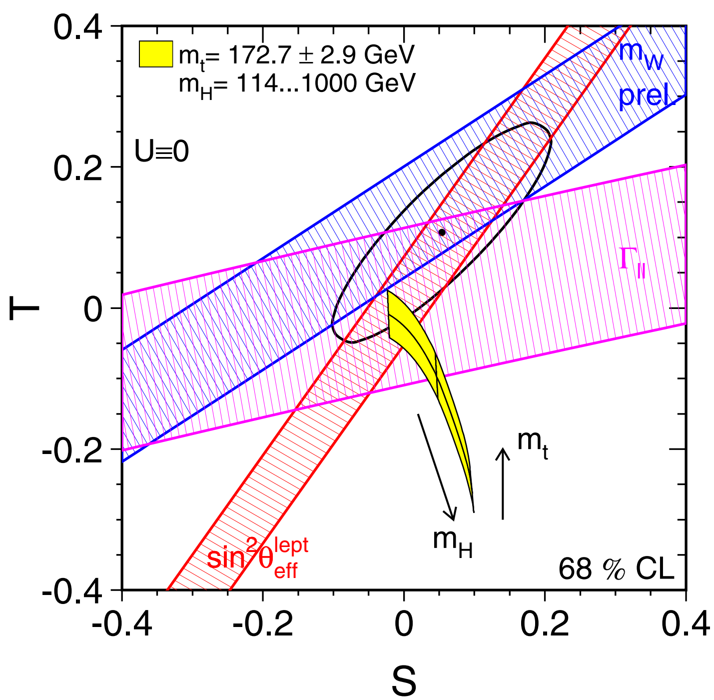
Implication of 10^{-3} accuracy to BSM: *a rule of thumb*

New particle mass at scale Λ

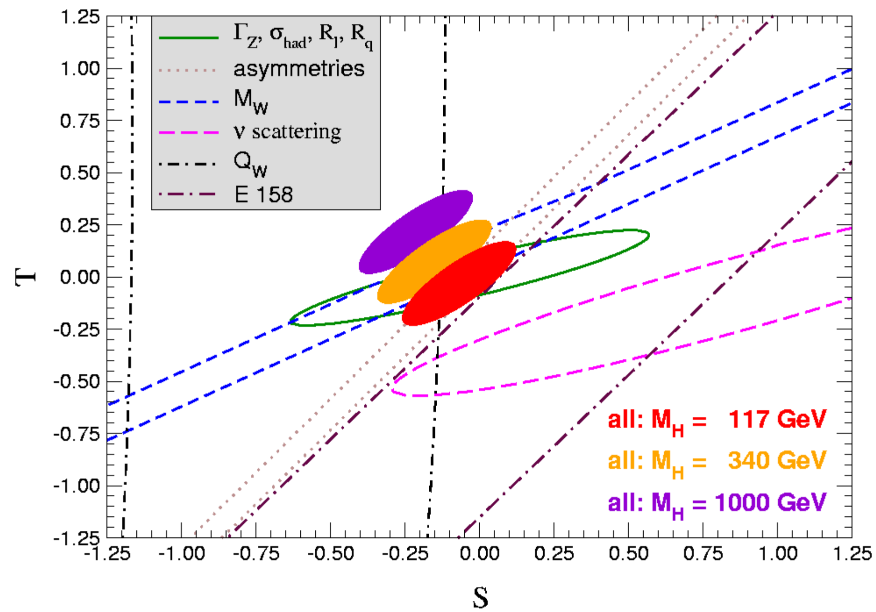
Three categories of BSM scenarios:

- New particle(s) contributing to EW physics at *one-loop level* through *non-decoupling* effects in the gauge boson vacuum polarization functions:
$$\frac{e^2}{(4\pi)^2} = \frac{\alpha}{4\pi} \simeq 10^{-3}$$
e.g., technicolor, heavy 4th generation, $\cdots$
In order to parametrize new physics effects in this class of models, we use
$$(S, T, U)$$
Peskin and Takeuchi, PRL65 (1990) 964
or
$$(\epsilon_1, \epsilon_2, \epsilon_3)$$
Alterelli and Barbieri, PLB253 (1991) 161
see also: Hagiwara, Matsumoto, Haidt and Kim, Z.Phys.C64 (1994) 559.

S-T plot



http://lepewwg.web.cern.ch/LEPEWWG/



Erler and Langacker, in RPP2006

- New particle(s) contributing to EW physics at *tree level*:

$$\frac{M_Z^2}{\Lambda^2} \sim 10^{-3}$$

e.g., Z' models, Higgsless models, little Higgs models, $\cdots$
Precision EW measurements are sensitive to new physics at

$$\Lambda \sim 3\text{TeV}.$$

There is no simple parametrization to describe the effects of every type of new physics in this class, however.

Recent proposal of parameters applicable to "universal" models (e.g., Higgsless models, little Higgs models):

$$(\hat{S}, \hat{T}, W, Y)$$

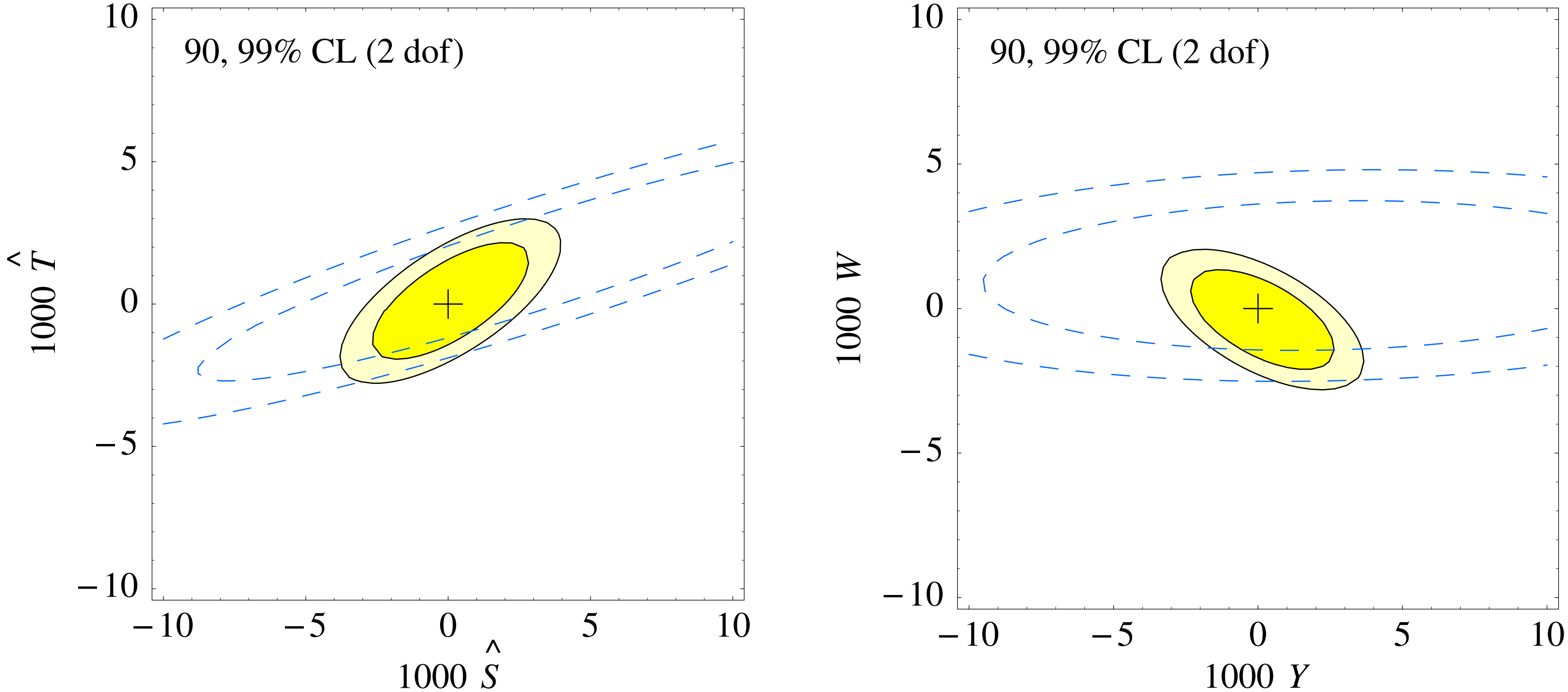
Barbieri et al. hep-ph/0405042

or

$$(S, T, \Delta\rho, \delta)$$

Chivukula et al. hep-ph/0408262

"Universal" model:



R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, NPB703 (2004) 127.

- New particle contributes to EW physics at *one-loop level* through *decoupling* effects:
$$\frac{1}{(4\pi)^2}\frac{M_Z^2}{\Lambda^2} \sim 10^{-3}.$$
e.g., TeV scale SUSY, little Higgs models with T-parity, $\cdots$
Precision EW measurements are sensitive to new physics at
$$\Lambda \sim 300\text{GeV}$$

Plan of this talk

§.0 **電弱精密測定の力**

§.1 **電弱標準理論の復習**

§.2 **デカップリング定理**

§.3 S-T **フィット**

§.4 **ヒッグス質量への制限**

§.5 **電弱カイラル摂動論**

§.6 “Universal” non-oblique corrections

§.7 **まとめ**

§.1 電弱標準理論 (WS 模型) の復習

WS model is a chiral $SU(2)_W \times U(1)_Y$ gauge theory:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \qquad a = 1, 2, 3$$

with

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g_W \epsilon^{abc} W^b_\mu W^c_\nu, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu .$$

Interaction with quarks and leptons:

$$\mathcal{L}_{\text{int}} = g_W J^{a\mu}_W W^a_\mu + g_Y J^\mu_Y B_\mu, \qquad J^{a\mu}_W = \sum_\psi \bar{\psi} I_a \gamma^\mu \psi, \quad J^\mu_Y = \sum_\psi \bar{\psi} Y \gamma^\mu \psi .$$

ψ	ℓ_L	e_R	q_L	u_R	d_R
I_a	$\frac{\tau_a}{2}$	0	$\frac{\tau_a}{2}$	0	0
Y	$-\frac{1}{2}$	-1	$\frac{1}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$

$$\ell_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

We need Higgs field ϕ so as to make the weak gauge bosons massive:

$$\phi = \begin{pmatrix} \varphi_+ \\ \varphi_0 \end{pmatrix}, \qquad \begin{array}{l} \text{weak } SU(2)_W \text{ doublet} \\ \text{weak hypercharge } Y = 1/2 \end{array}$$

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi), \qquad V(\phi) = \lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2,$$

with

$$D_\mu \phi = \left(\partial_\mu + i g_W \frac{\tau^a}{2} W_\mu^a + i g_Y \frac{1}{2} B_\mu \right) \phi.$$

Thanks to the wine bottle shape of the Higgs potential $V(\phi)$, Higgs field acquires its VEV:

$V(\phi) \simeq$  $\Rightarrow \qquad \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$

and breaks $SU(2)_W \times U(1)_Y$ into $U(1)_Q$ spontaneously.

Neutral current

W_μ^3 and B^μ mix with each other:

$$\mathcal{L}_{\text{mass}} = \frac{1}{8}(W_\mu^3 \;\, B_\mu) \begin{pmatrix} g_W^2 v^2 & -g_W g_Y v^2 \\ -g_W g_Y v^2 & g_Y^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix},$$

Mass diagonalization:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \qquad s = \sin\theta_W\,, \quad c = \cos\theta_W\,,$$

with Weinberg angle θ_W being given by

$$\sin\theta_W = \frac{g_Y}{\sqrt{g_W^2 + g_Y^2}}, \qquad \cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g_Y^2}}.$$

Mass eigenvalues

$$M_\gamma^2 = 0, \qquad M_Z^2 = \frac{g_W^2 + g_Y^2}{4} v^2.$$

Z and photon interactions with quarks/leptons:

$$\mathcal{L}_{\text{int}}^{\text{NC}} = e \sum_{\psi} \bar{\psi} \mathcal{Q} \gamma^{\mu} \psi A_{\mu} + \frac{e}{sc} \sum_{\psi} \bar{\psi} (I_3 - s^2 \mathcal{Q}) \gamma^{\mu} \psi Z_{\mu},$$

with

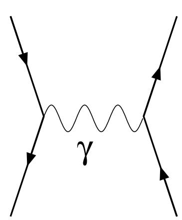
$$e^2 = \frac{g_W^2 g_Y^2}{g_W^2 + g_Y^2}, \qquad \frac{e^2}{s^2 c^2} = g_W^2 + g_Y^2, \qquad M_Z^2 = \frac{e^2}{s^2 c^2} \frac{v^2}{4}.$$

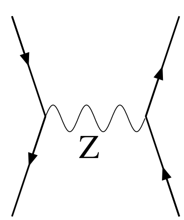
$$\mathcal{Q} \equiv I_3 + Y$$

(vector-like)

ψ	ν_L	e_L	e_R	u_L	u_R	d_L	d_R
$\mathcal{Q}$	0	-1	-1	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$

Neutral current $f\bar{f} \to f'\bar{f}'$ amplitude:

$\mathcal{M}_{\text{NC}} =$ 

$+$ 

$$= e^2 \frac{\mathcal{Q}\mathcal{Q}'}{k^2} + \frac{e^2}{s^2 c^2} \frac{(I_3 - s^2 \mathcal{Q})(I_3' - s^2 \mathcal{Q}')}{k^2 - M_Z^2}.$$

Charged current

W boson mass term:

$$\mathcal{L}_{\text{mass}} = \frac{g_W^2 v^2}{4} W_\mu^+ W^{-\mu}, \qquad W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2).$$

Mass of W:

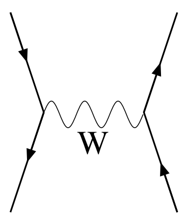
$$M_W^2 = \frac{g_W^2}{4} v^2 = \frac{e^2}{4s^2} v^2, \qquad \frac{e^2}{s^2} = g_W^2.$$

W boson interaction with quarks/leptons:

$$\mathcal{L}_{\text{int}}^{\text{CC}} = \frac{1}{\sqrt{2}} \frac{e}{s} \sum_\psi \bar{\psi} I_- \gamma^\mu \psi W_\mu^+ + \text{h.c.}, \qquad I_\pm \equiv I_1 \mp i I_2.$$

Charged current $f\bar{f} \to f'\bar{f}'$ amplitude:

$$\mathcal{M}_{\text{CC}} =$$



$$= \frac{e^2}{s^2} \frac{(I_+ I'_- + I_- I'_+)/2}{k^2 - M_W^2}.$$

Custodial $SU(2)$ symmetry

Low energy four-fermion couplings from W and Z exchanges:

$$4\sqrt{2}G_{\rm CC} = \frac{e^2}{s^2}\frac{1}{M_W^2} = \frac{4}{v^2}, \qquad 4\sqrt{2}G_{\rm NC} = \frac{e^2}{s^2c^2}\frac{1}{M_Z^2} = \frac{4}{v^2}.$$

$$\rho \equiv \frac{G_{\rm NC}}{G_{\rm CC}} = 1. \qquad \text{(doublet Higgs の特徴)}$$

$\rho = 1$ **を保証する物理はなにか**?

Higgs Lagrangian can be rewritten as

$$\mathcal{L}_{\rm Higgs} = \frac{1}{4}\text{tr}\left((D_\mu\Phi)^\dagger(D_\mu\Phi)\right) - V(\Phi),$$

with $$\Phi = \sqrt{2}\left(\tilde{\phi}\phi\right) = \sqrt{2}\begin{pmatrix} \varphi_0^* & \varphi_+ \\ -\varphi_+^* & \varphi_0 \end{pmatrix}, \qquad \tilde{\phi} \equiv i\tau_2\phi^*.$$

$$D_\mu\Phi = \partial_\mu\Phi + ig_W\frac{\tau_a}{2}W_\mu^a\Phi - ig_Y\Phi\frac{\tau_3}{2}B_\mu.$$

Symmetry of the Higgs Lagrangian is enhanced to $SU(2)_W \times SU(2)_R$ in the $g_Y \to 0$ limit:

$$\Phi \to U_L \Phi U_R^\dagger, \qquad U_L \in SU(2)_W, \quad U_R \in SU(2)_R$$

VEV of Higgs breaks $SU(2)_W \times SU(2)_R$ symmetry to diagonal $SU(2)_C$:

$$SU(2)_W \times SU(2)_R \to SU(2)_C,$$

$$\langle\Phi\rangle = \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix}, \qquad \langle\Phi\rangle \to \langle U_C \Phi U_C^\dagger\rangle = U_C \langle\Phi\rangle U_C^\dagger = \langle\Phi\rangle, \quad U_C \in SU(2)_C.$$

$SU(2)_C$: **カストディアル** $SU(2)$ **対称性のもとで、** W, Z **に吸収される** NG **ボソンは** triplet **となる**

It will turn out that the cutdodial $SU(2)$ symmetry is a useful concept in the parametrization of new physics in the precision EW tests.

Number of free parameters

Fermi coupling G_F is determined as

$$4\sqrt{2}G_F = \frac{e^2}{s^2}\frac{1}{M_W^2} = \frac{4}{v^2}.$$

At the tree-level, structure of fermion scattering amplitude is determined completely once *three parameters* (e, s, G_F) are all fixed:

- Charged current process

$$-\mathcal{M}_{\mathrm{CC}} = \frac{(I_+ I'_- + I_- I'_+)/2}{-\dfrac{s^2}{e^2}k^2 + \dfrac{1}{4\sqrt{2}G_F}}, \qquad M_W^2 = \frac{e^2}{s^2}\frac{1}{4\sqrt{2}G_F},$$

- Neutral current process

$$-\mathcal{M}_{\mathrm{NC}} = e^2\frac{\mathcal{Q}\mathcal{Q}'}{-k^2} + \frac{(I_3 - s^2\mathcal{Q})(I'_3 - s^2\mathcal{Q}')}{-\dfrac{s^2c^2}{e^2}k^2 + \dfrac{1}{4\sqrt{2}G_F}}, \qquad M_Z^2 = \frac{e^2}{s^2c^2}\frac{1}{4\sqrt{2}G_F}.$$

What is $\sin\theta_W$? (extraction of $\sin\theta_W$ from EW observables)

- (e, M_W, M_Z) scheme:

$$s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}, \qquad c_W^2 \equiv 1 - s_W^2$$

- (e, G_F, M_W) scheme:

$$s_{M_W}^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_W^2}, \qquad c_{M_W}^2 \equiv 1 - s_{M_W}^2$$

- (e, G_F, M_Z) scheme:

$$s_{M_Z}^2 c_{M_Z}^2 \equiv \frac{e^2}{4\sqrt{2} G_F M_Z^2}, \qquad c_{M_Z}^2 \equiv 1 - s_{M_Z}^2$$

These definitions are all equivalent at the tree-level.

§.2 デカップリング定理

我々のやりたいこと:

Hunting new physics through the precision tests of the standard model

悪い知らせ: *decoupling theorem*

T. Appelquist and J. Carazzone, PRD11 (1975) 2856.

If the new physics remains perturbative in the heavy particle limit, all effects of the heavy particle are suppressed by powers of the heavy particle mass.

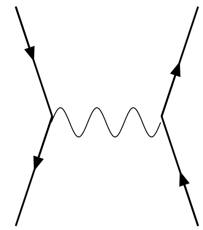
よい知らせ: *violation of decoupling theorem*

The standard model is a spontaneously broken chiral gauge theory. There is a class of new physics scenarios in which heavy particles' masses are proportional to their couplings. (e.g., technicolor, heavy 4th generation, $\cdots$)

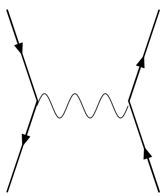
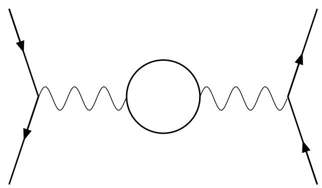
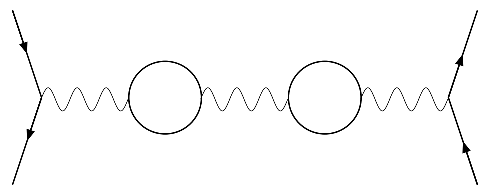
Question: How can we parametrize such non-decoupling effects? How many parameter do we have?

Peskin-Takeuchi parameters S, T, U for oblique correction.

Decoupling theorem in QED

Born amplitude of $f\bar{f} \to f'\bar{f}'$ scattering in QED:  $= \mathcal{Q}\dfrac{e_0^2}{k^2}\mathcal{Q}'.$

Consider new particle of mass M_{new} contributing to the photon vacuum polarization function ($oblique$ correction)

 $+$  $+$  $+\cdots$

$$= \mathcal{Q}\frac{e_0^2}{k^2}\mathcal{Q}' + \mathcal{Q}\frac{e_0^2}{k^2}\Pi_{\text{new}}\frac{e_0^2}{k^2}\mathcal{Q}' + \mathcal{Q}\frac{e_0^2}{k^2}\Pi_{\text{new}}\frac{e_0^2}{k^2}\Pi_{\text{new}}\frac{e_0^2}{k^2}\mathcal{Q}' + \cdots$$

$$= \mathcal{Q}\frac{e_0^2}{k^2 - e_0^2\Pi_{\text{new}}}\mathcal{Q}' = \frac{\mathcal{Q}\mathcal{Q}'}{\dfrac{1}{e_0^2}k^2 - \Pi_{\text{new}}}$$

Radiative corrections from *known* physics are ignored for simplicity.

QED gauge invariance

$$\Pi^{\mu\nu}_{\rm new}(k^2) = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\Pi_{\rm new}(k^2) = \left(g^{\mu\nu}k^2 - k^\mu k^\nu\right)\widetilde{\Pi}_{\rm new}(k^2)$$

reads

$$\Pi_{\rm new}(0) = 0.$$

Simple consideration based on the mass dimension

$$\text{dim}\widetilde{\Pi}_{\rm new}(k^2) = 0$$

suggests $\widetilde{\Pi}_{\rm new}$ scales like

$$\widetilde{\Pi}_{\rm new}(0) \propto (M_{\rm new})^0$$

Radiative correction $\widetilde{\Pi}_{\rm new}(k^2)$ seems to be nonvanishing even in the $M_{\rm new} \to \infty$ limit.

We should be careful about the renormalization.

Acutually, the charge renormalization procedure

$$\frac{1}{e^2} = \frac{1}{e_0^2} - \widetilde{\Pi}_{\rm new}(k^2 = 0),$$

absorbs the nonvanishing $\widetilde{\Pi}_{\rm new}(0)$. Improved Born $f\bar{f} \to f'\bar{f}'$ scattering amplitude can then be written as

$$\mathcal{M}_{\rm QED} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{e_0^2} - \widetilde{\Pi}_{\rm new}(k^2)\right) k^2} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{e^2} - k^2\widetilde{\Pi}'_{\rm new}(0) + \mathcal{O}(k^4)\right) k^2},$$

with

$$\widetilde{\Pi}_{\rm new}(k^2) = \widetilde{\Pi}_{\rm new}(0) + k^2\widetilde{\Pi}'_{\rm new}(0) + \cdots.$$

Simple analysis based on mass dimension:

$$\text{dim}\widetilde{\Pi}'_{\rm new} = -2$$

suggests

$$\widetilde{\Pi}'_{\rm new} \sim \frac{1}{M^2_{\rm new}},$$

with $M^2_{\rm new}$ being the mass scale of new physics.

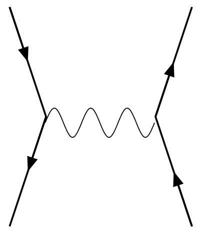
New physics thus decouples from the low energy QED $f\bar{f} \to f'\bar{f}'$ scattering amplitude in $M_{\rm new} \to \infty$ limit,

$$\mathcal{M}_{\rm QED} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{e^2} + \mathcal{O}\left(\frac{k^2}{M^2_{\rm new}}\right)\right) k^2},$$

if we write the amplitude in terms of appropriately renormalized couplings.

Appelquist-Carazzone decoupling theorem

Spontaneously broken $U(1)$ gauge theory

Born amplitude:  $= \mathcal{Q}\dfrac{g_0^2}{k^2 - g_0^2 v_0^2}\mathcal{Q}'.$

New physics contribution to the vacuum polarization $\Pi_{\rm new}$.

Improved Born amplitude

$$\mathcal{M} = \frac{\mathcal{Q}\mathcal{Q}'}{\frac{1}{g_0^2}k^2 - v_0^2 - \Pi_{\rm new}(k^2)}.$$

The $U(1)$ gauge invariance is broken spontanesously

$$\Pi_{\rm new}(0) \neq 0.$$

We define

$$\Pi_{\rm new}(k^2) = \Pi_{\rm new}(0) + k^2 \widetilde{\Pi}_{\rm new}(k^2)$$

Non-zero $\Pi_{\rm new}(0)$ and $\widetilde{\Pi}_{\rm new}(0)$ are absorbed into the renormalization of v and g, respectively:

$$v^2 = v_0^2 + \Pi_{\rm new}(0), \qquad \frac{1}{g^2} = \frac{1}{g_0^2} - \widetilde{\Pi}_{\rm new}(0).$$

Remaining correction $\widetilde{\Pi}'_{\rm new}(0)$ in

$$\mathcal{M} = \frac{\mathcal{Q}\mathcal{Q}'}{\left(\frac{1}{g^2} - k^2 \widetilde{\Pi}'_{\rm new}(0) + \cdots\right) k^2 - v^2}$$

behaves as

$$\widetilde{\Pi}'_{\rm new}(0) \sim \frac{1}{M_{\rm new}^2}$$

and decouples from the low energy amplitude in the $M_{\rm new} \to \infty$ limit.

Violation of decoupling theorem in EW physics

$$SU(2)_W \times U(1)_Y \to U(1)_Q$$

Number of vacuum polarization functions (*oblique corrections*):

- Charged current: $$\Pi_{11}(k^2) = \Pi_{22}(k^2)$$
- Neutral current: $$\Pi_{33}(k^2), \quad \Pi_{3Q}(k^2), \quad \Pi_{QQ}(k^2).$$

Thanks to the unbroken QED gauge invariance

$$\begin{aligned}
\Pi_{11}^{\rm new}(k^2) &= \Pi_{11}^{\rm new}(0) + k^2\widetilde{\Pi}_{11}^{\rm new}(0) + \mathcal{O}(\frac{k^4}{M_{\rm new}^2}) \\
\Pi_{33}^{\rm new}(k^2) &= \Pi_{33}^{\rm new}(0) + k^2\widetilde{\Pi}_{33}^{\rm new}(0) + \mathcal{O}(\frac{k^4}{M_{\rm new}^2}) \\
\Pi_{3Q}^{\rm new}(k^2) &= \qquad\qquad\quad k^2\widetilde{\Pi}_{3Q}^{\rm new}(0) + \mathcal{O}(\frac{k^4}{M_{\rm new}^2}) \\
\Pi_{QQ}^{\rm new}(k^2) &= \qquad\qquad\quad k^2\widetilde{\Pi}_{QQ}^{\rm new}(0) + \mathcal{O}(\frac{k^4}{M_{\rm new}^2})
\end{aligned}$$

- 6 seemingly non-decoupling degree of freedoms:

$$\Pi_{11}^{\rm new}(0), \Pi_{33}^{\rm new}(0), \widetilde{\Pi}_{11}^{\rm new}(0), \widetilde{\Pi}_{33}^{\rm new}(0), \widetilde{\Pi}_{3Q}^{\rm new}(0), \widetilde{\Pi}_{QQ}^{\rm new}(0)$$

- 3 renormalization:

$$g_W\,, \quad g_Y\,, \quad v, \qquad\qquad \text{or } (e, s, G_F)$$

- $6-3=3$ non-decoupling parameters left unabsorbed after the renormalization (Peskin-Takeuchi parameters):

$$\begin{aligned}
\alpha S &= 4e^2\left(\widetilde{\Pi}_{33}^{\rm new}(0)-\widetilde{\Pi}_{3Q}^{\rm new}(0)\right), \\
\alpha T &= 4\sqrt{2}G_F\left(\Pi_{11}^{\rm new}(0)-\Pi_{33}^{\rm new}(0)\right), \\
\alpha U &= 4e^2\left(\widetilde{\Pi}_{11}^{\rm new}(0)-\widetilde{\Pi}_{33}^{\rm new}(0)\right).
\end{aligned}$$

Fermion scattering amplitude

- Charged current process

$$-\mathcal{M}_{\rm CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{-\left(\frac{s^2}{e^2} - \frac{S+U}{16\pi}\right) k^2 + \frac{1}{4\sqrt{2} G_F}}$$

- Neutral current process

$$-\mathcal{M}_{\rm NC} = e^2 \frac{\mathcal{Q}\mathcal{Q}'}{-k^2} + \frac{(I_3 - s^2 \mathcal{Q})(I'_3 - s^2 \mathcal{Q}')}{-\left(\frac{s^2 c^2}{e^2} - \frac{S}{16\pi}\right) k^2 + \frac{1}{4\sqrt{2} G_F} \left(1 - \frac{e^2}{4\pi} T\right)}$$

What is $\sin\theta_W$?

- (e, M_W, M_Z) scheme:

$$s_W^2 \equiv 1 - \frac{M_W^2}{M_Z^2}, \qquad c_W^2 \equiv 1 - s_W^2$$

- (e, G_F, M_W) scheme:

$$s_{M_W}^2 \equiv \frac{e^2}{4\sqrt{2}G_F M_W^2}, \qquad c_{M_W}^2 \equiv 1 - s_{M_W}^2$$

- (e, G_F, M_Z) scheme:

$$s_{M_Z}^2 c_{M_Z}^2 \equiv \frac{e^2}{4\sqrt{2}G_F M_Z^2}, \qquad c_{M_Z}^2 \equiv 1 - s_{M_Z}^2$$

$$
\begin{aligned}
M_W^2 &= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2}\left[1+\frac{1}{4s^2}(\alpha S+\alpha U)\right], \\
M_Z^2 &= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2c^2}\left[1+\frac{1}{4s^2c^2}\alpha S-\alpha T\right], \qquad \alpha\equiv\frac{e^2}{4\pi}.
\end{aligned}
$$

$$
\begin{aligned}
s_W^2 &= s^2+\Delta_W, \qquad \Delta_W=\frac{\alpha}{4}S-c^2\alpha T-\frac{c^2}{s^2}\frac{\alpha}{4}U, \\
s_{M_W}^2 &= s^2+\Delta_{M_W}, \qquad \Delta_{M_W}=-\frac{\alpha}{4}S-\frac{\alpha}{4}U, \\
s_{M_Z}^2 &= s^2+\Delta_{M_Z}, \qquad \Delta_{M_Z}=\frac{1}{c^2-s^2}\left[-\frac{\alpha}{4}S+s^2c^2\alpha T\right].
\end{aligned}
$$

$$
s^2\neq s_W^2\neq s_{M_W}^2\neq s_{M_Z}^2
$$

Lesson: We need to be careful about the definition of $\sin\theta_W$ under the presence of S, T, U (or at the loop-level).

Effects of heavy fermion loop

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \qquad U_R, \quad D_R$$

Heavy U and D $\Rightarrow$ Large Yukawa coupling

violation of decoupling theorem

$$S = \frac{N}{6\pi}\left[1 - 2Y_{Q_L}\ln\frac{m_U^2}{m_D^2}\right], \qquad Y_{Q_L}: \text{ weak hypercharge of } Q_L$$

$$T = \frac{N}{16\pi s^2 c^2 M_Z^2}\left[m_U^2 + m_D^2 - \frac{2m_U^2 m_D^2}{m_U^2 - m_D^2}\ln\frac{m_U^2}{m_D^2}\right],$$

$$U = \frac{N}{6\pi}\left[-\frac{5m_U^4 - 22m_U^2 m_D^2 + 5m_D^4}{3(m_U^2 - m_D^2)^2} + \frac{m_U^6 - 3m_U^4 m_D^2 - 3m_U^2 m_D^4 + m_D^4}{(m_U^2 - m_D^2)^3}\ln\frac{m_U^2}{m_D^2}\right].$$

If U and D are almost degenrated,

$$|\Delta m| \ll \hat{m}, \qquad \Delta m \equiv m_U - m_D, \qquad \hat{m} \equiv \frac{m_U + m_D}{2}$$

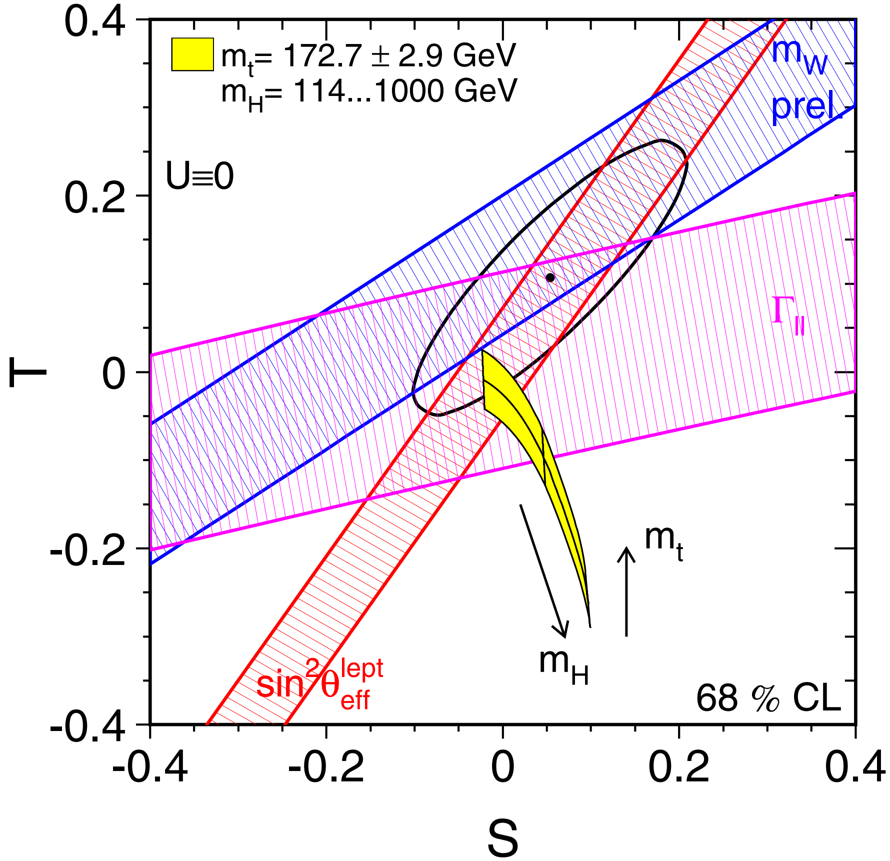
we find

$$S \simeq \frac{N}{6\pi}, \quad T \simeq \frac{N}{12\pi s^2 c^2} \frac{(\Delta m)^2}{M_Z^2}, \quad U \simeq \frac{2N}{15\pi} \frac{(\Delta m)^2}{\hat{m}^2}.$$

Note:

- If custodial $SU(2)$ symmetry is exact, $\Delta m = 0$ and thus $T = U = 0$. Nonzero T (and U) should be regarded as a consequence of the custodial $SU(2)$ violation.
- The size of U is extremely suppressed for $(\Delta m)^2 \ll \hat{m}^2$.
- Sizable contribution to T is possible for $(\Delta m)^2 \sim M_Z^2 \ll \hat{m}^2$.
- Degenerated heavy 4th generation: $S = \dfrac{4}{6\pi} \simeq 0.21$, $T = 0$, $U = 0$

§.3 S-T フィット LEPEWWG2005



http://lepewwg.web.cern.ch/LEPEWWG/

- $U = 0$ is assumed.
- $m_t^{\mathrm{ref}} = 175$GeV
- $m_H^{\mathrm{ref}} = 150$GeV

The value of $\sin^2\theta_{\rm eff}^{\rm lept}$ is extracted from asymmetries on Z-pole:

$e^-e^+ \to \ell^-\ell^+$ forward-backward asymmetry:

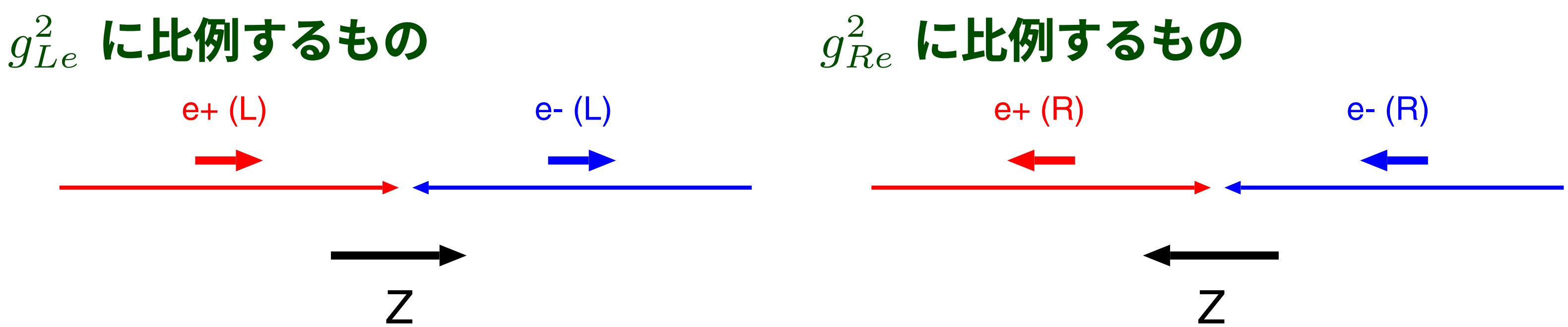
$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_\ell, \qquad \mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}$$

$$\frac{g_{V\ell}}{g_{A\ell}} = \frac{-\frac{1}{4} - Q_\ell \sin^2\theta_{\rm eff}^{\rm lept}}{-\frac{1}{4}} = 1 - 4\sin^2\theta_{\rm eff}^{\rm lept},$$

$$\begin{aligned}
\sin^2\theta_{\rm eff}^{\rm lept} &= s^2 + (\text{SM correction}) \\
&= s_{M_Z}^2\left(1 + \frac{1}{4s^2(c^2 - s^2)}\alpha S - \frac{c^2}{c^2 - s^2}\alpha T + (\text{SM correction})\right) \\
&= s_{M_Z}^2 \times (1 + 2.01 \times \alpha S - 1.43 \times \alpha T + (\text{SM correction}))
\end{aligned}$$

電子の質量を無視して考察する

e^-e^+ 衝突による Z の生成

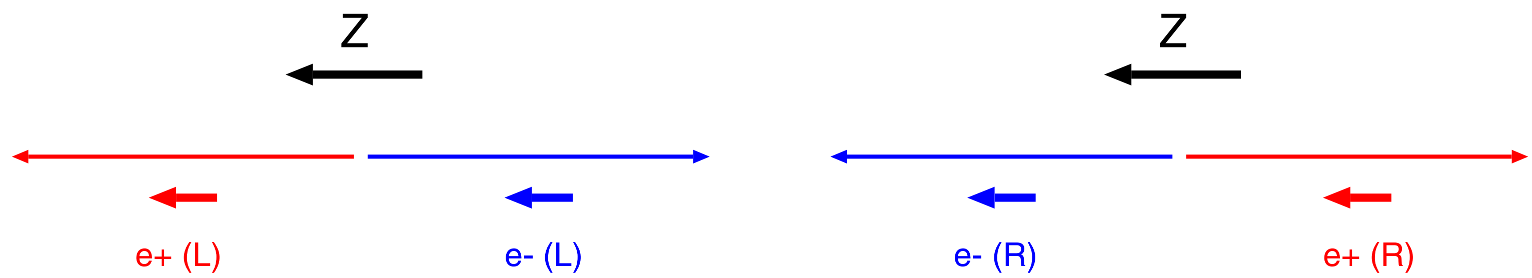


$$g_{Le}^2 \neq g_{Re}^2$$
$$\Downarrow$$

生成された Z は偏極電子ビームを使わなくても自然に偏極している

偏極 Z 粒子の $e^- e^+$ への崩壊

g_{Le}^2 に比例するもの　　　　g_{Re}^2 に比例するもの



$$g_{Le}^2 > g_{Re}^2$$

$$\Downarrow$$

e^- は Z の偏極とは逆向きに出てきやすい

$$\Downarrow$$

FB asymmetry

$\Gamma(Z \to \ell^+\ell^-) = 83.985(86)\mathrm{MeV}$

$Z\bar{e}_L e_L$ coupling: $(I_3 = -1/2,\ \mathcal{Q} = -1)$

$$g_{Le}^2 = \frac{e^2}{s^2c^2}\left(-\frac{1}{2} + s^2\right)^2\left(1 + \frac{\alpha}{4s^2c^2}S\right)$$

$Z\bar{e}_R e_R$ coupling: $(I_3 = 0,\ \mathcal{Q} = -1)$

$$g_{Re}^2 = \frac{e^2}{s^2c^2}\left(0 + s^2\right)^2\left(1 + \frac{\alpha}{4s^2c^2}S\right)$$

$$\Gamma(Z \to \ell^+\ell^-) \propto g_{L\ell}^2 + g_{R\ell}^2$$

$$= \quad \frac{e^2}{s^2c^2}\left((-\frac{1}{2} + s^2)^2 + s^4\right)\left(1 + \frac{1}{4s^2c^2}\alpha S\right)$$

$$= \quad \frac{e^2}{s_{M_Z}^2 c_{M_Z}^2}\left((-\frac{1}{2} + s_{M_Z}^2)^2 + s_{M_Z}^4\right)(1 - 0.281 \times \alpha S + 1.20 \times \alpha T)$$

M_W^2

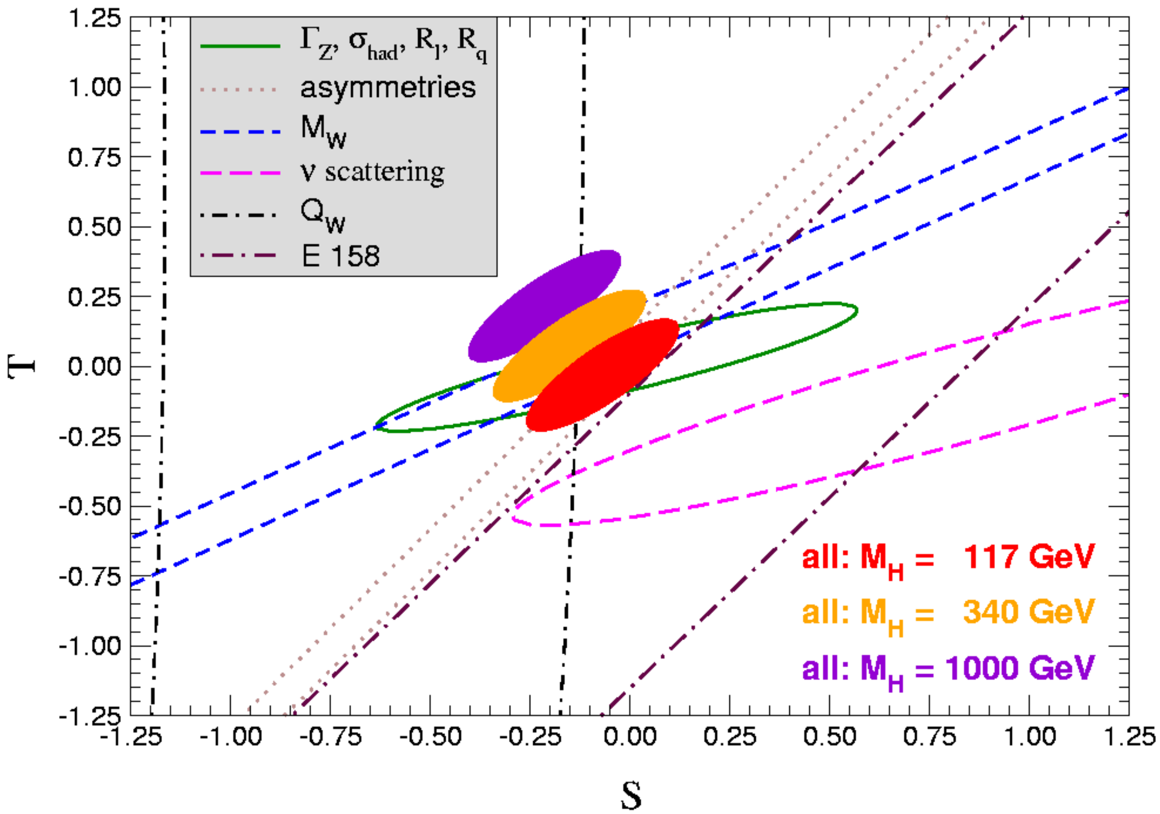
$$
\begin{aligned}
M_W^2 &= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2_{M_W}} \\
&= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2_{M_Z}}\left[1-\frac{1}{s^2}(\Delta_{M_W}-\Delta_{M_Z})+\text{(SM correction)}\right] \\
&= \frac{1}{4\sqrt{2}G_F}\frac{e^2}{s^2_{M_Z}}\left[1-0.930\times\alpha S+1.43\times\alpha T+1.08\times\alpha U+\text{(SM corr.)}\right]
\end{aligned}
$$

SM ambiguities

S, T are defined as deviations from the SM: we should be careful...

$$
\begin{aligned}
S &\simeq \frac{1}{12\pi}\ln\frac{M_H^2}{M_{H,\mathrm{ref}}^2} + \frac{1}{6\pi}\ln\frac{m_t^2}{m_{t,\mathrm{ref}}^2} \\
T &\simeq -\frac{3}{16\pi c^2}\ln\frac{M_H^2}{M_{H,\mathrm{ref}}^2} + \frac{3}{16\pi s^2 c^2}\frac{m_t^2 - m_{t,\mathrm{ref}}^2}{M_Z^2} \\
U &\simeq \frac{1}{2\pi}\ln\frac{m_t^2}{m_{t,\mathrm{ref}}^2}
\end{aligned}
$$

S-T plot of Erler-Langacker review in RPP2006



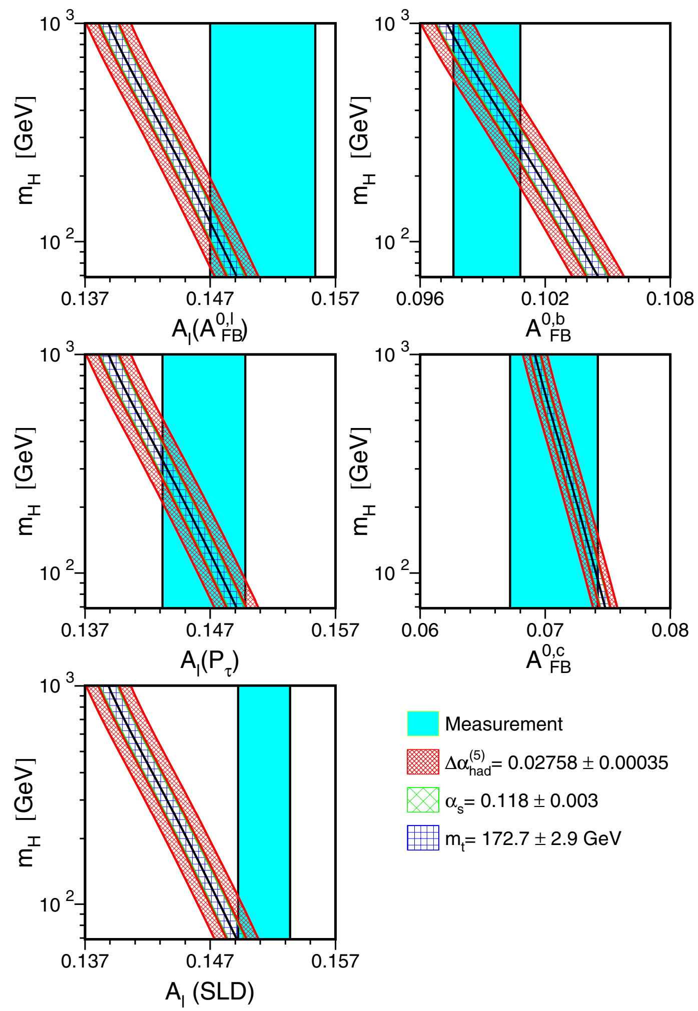
$$M_H^{\mathrm{ref}} = 117\mathrm{GeV}$$
$$S = -0.13 \pm 0.10$$
$$T = -0.13 \pm 0.11$$
$$U = 0.20 \pm 0.12$$

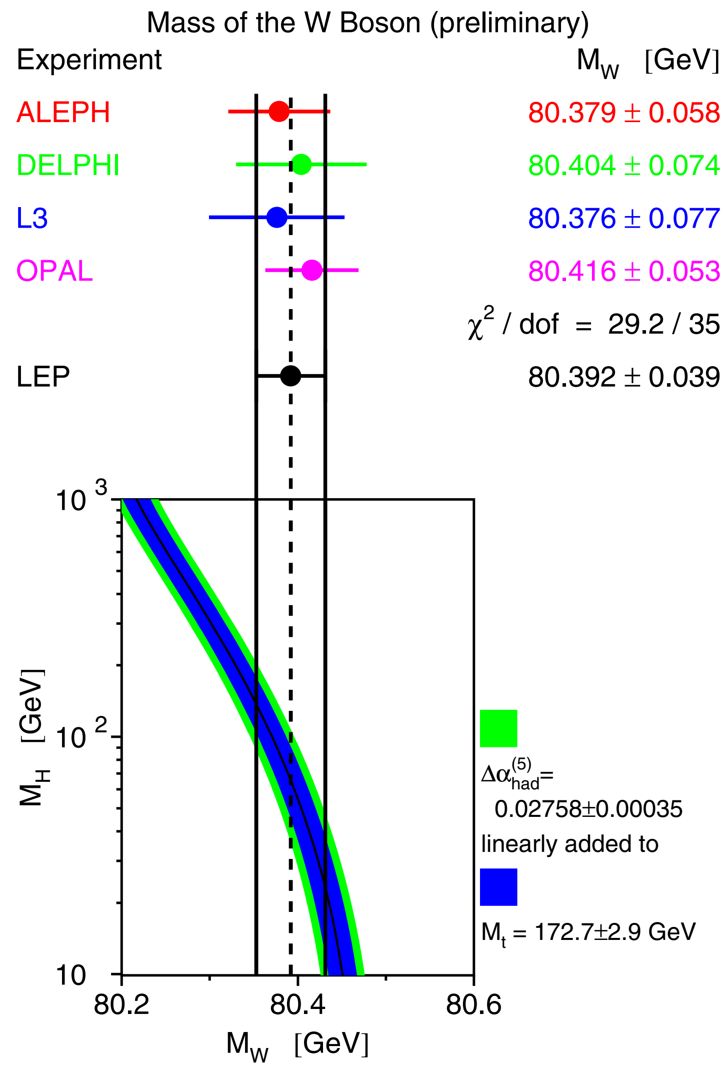
$$M_H^{\mathrm{ref}} = 300\mathrm{GeV}$$
$$S = -0.21 \pm 0.10$$
$$T = -0.04 \pm 0.11$$
$$U = 0.21 \pm 0.12,$$

90%CL

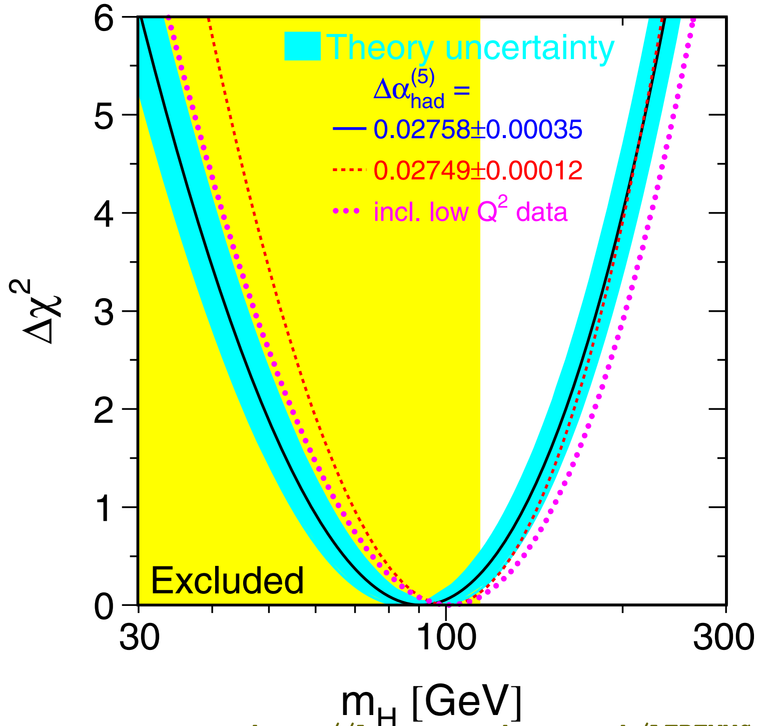
Erler and Langacker, in RPP2006

§.4 ヒッグス質量への制限





http://lepewwg.web.cern.ch/LEPEWWG/



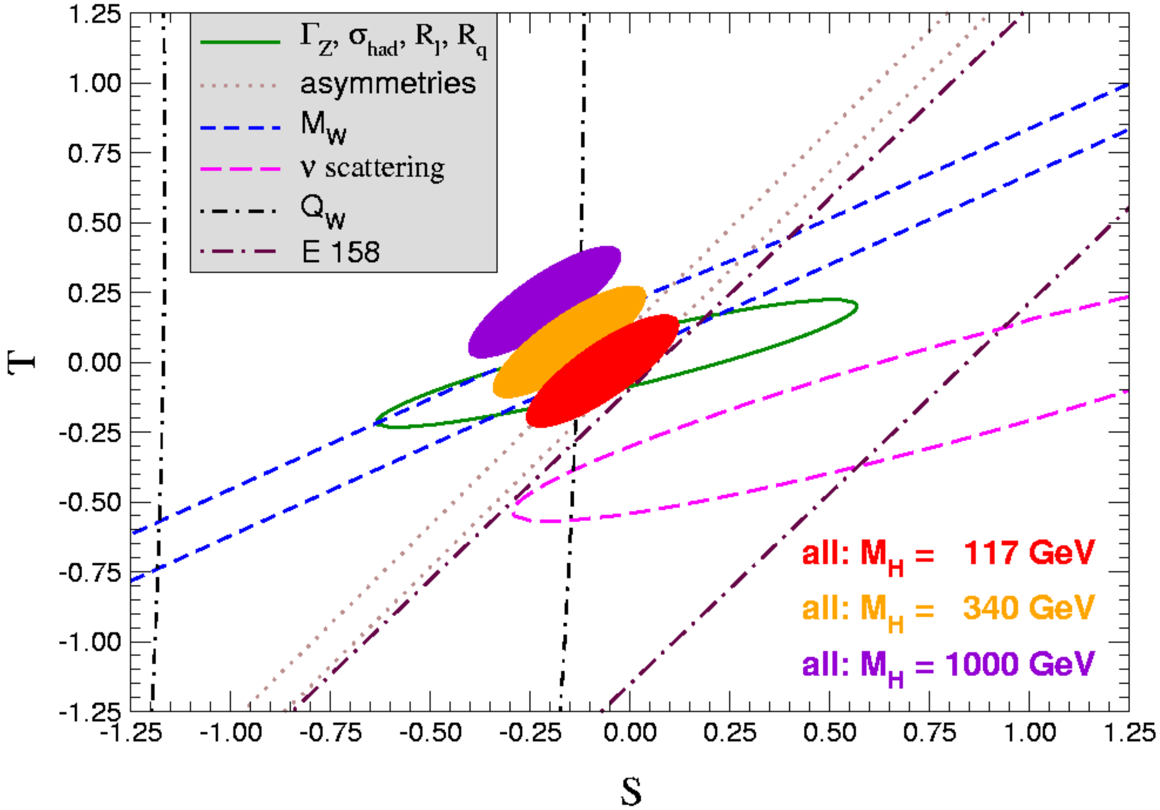
http://lepewwg.web.cern.ch/LEPEWWG/

$M_H \leq 199$GeV (95% CL)

Heavy Higgs looks inconsistent with precision data!

We need to be careful: this bound is relaxed significantly if there exists other positive (negative) contribution to the T-parameter (S-parameter). Higgs mass heavier than this bound indicates the presence of BSM.

S-T plot of Erler-Langacker review in RPP2006



$$S = -0.13 \pm 0.10$$
$$T = -0.13 \pm 0.11$$
$$U = 0.20 \pm 0.12$$

for $M_H = 117$GeV

90%CL

Erler and Langacker, in RPP2006

$$S = -0.21 \pm 0.10$$
$$T = -0.04 \pm 0.11$$
$$U = 0.21 \pm 0.12$$

for $M_H = 300$GeV

§.5 電弱カイラル摂動理論

Catalogue of *non-decoupling* corrections

Use of the gauged non-linear σ model

T. Appelquist and C. Bernard, PRD22 (1980) 200.

Taking the $M_H \to \infty$ limit, the original linear σ model Higgs field Φ is replaced by the non-linear σ model field U:

$$\Phi = \sqrt{2}\begin{pmatrix} \varphi_0^* & \varphi_+ \\ -\varphi_+^* & \varphi_0 \end{pmatrix} \to vU = v\exp\left(\frac{i\tau^a w^a}{v}\right),$$

with w^a being the NG bosons eaten by $W^\pm$, Z.

In this limit, the original Higgs Lagrangian is replaced by

$$\mathcal{L}_0 = \frac{1}{4}v^2 \mathrm{tr}[(D_\mu U)^\dagger (D^\mu U)] - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu},$$

where

$$D_\mu U = \partial_\mu U + ig_W \frac{\tau_a}{2}W^a_\mu U - ig_Y U\frac{\tau_3}{2}B_\mu .$$

Idea of Appelquist-Bernard paper

We can catalogue all possible non-decouplings effects as coefficients of operators upto dimension 4.

A. Longhitano, PRD22 (1980) 1166 ; NPB188 (1981) 118.
T. Appelquist and G.-H. Wu, PRD48 (1993) 3235.

List of CP even dimension 4 operators.

$$\begin{aligned}
\mathcal{L}_1 &\equiv \frac{1}{2}\alpha_1 g_W g_Y B_{\mu\nu}\mathrm{tr}(TW^{\mu\nu}), \\
\mathcal{L}_2 &\equiv i\alpha_2 g_Y B_{\mu\nu}\mathrm{tr}(T[V^\mu, V^\nu]), \\
\mathcal{L}_3 &\equiv i\alpha_3 g_W \mathrm{tr}(W_{\mu\nu}[V^\mu, V^\nu]), \\
\mathcal{L}_4 &\equiv \alpha_4[\mathrm{tr}(V_\mu V_\nu)]^2, \\
\mathcal{L}_5 &\equiv \alpha_5[\mathrm{tr}(V_\mu V^\mu)]^2,
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_6 &\equiv \alpha_6\mathrm{tr}(V_\mu V_\nu)\mathrm{tr}(TV^\mu)\mathrm{tr}(TV^\nu), \\
\mathcal{L}_7 &\equiv \alpha_7\mathrm{tr}(V_\mu V^\mu)\mathrm{tr}(TV_\nu)\mathrm{tr}(TV^\nu), \\
\mathcal{L}_8 &\equiv \frac{1}{4}\alpha_8 g_W^2[\mathrm{tr}(TW_{\mu\nu})]^2, \\
\mathcal{L}_9 &\equiv \frac{1}{2}i\alpha_9 g_W\mathrm{tr}(TW_{\mu\nu})\mathrm{tr}(T[V^\mu, V^\nu]), \\
\mathcal{L}_{10} &\equiv \frac{1}{2}\alpha_{10}[\mathrm{tr}(TV_\mu)\mathrm{tr}(TV_\nu)]^2, \\
\mathcal{L}_{11} &\equiv \alpha_{11} g\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(TV_\mu)\mathrm{tr}(V_\nu W_{\rho\lambda}).
\end{aligned}$$

where

$$T \equiv U\tau_3 U^\dagger, \quad V_\mu \equiv (D_\mu U)U^\dagger, \qquad \mathrm{dim}(\alpha_i) = 0.$$

One additional dimension 2 operator

$$\mathcal{L}'_1 \equiv \frac{1}{4}\beta_1 v^2 \left[\text{tr}(TV_\mu)\right]^2 ,$$

which violates custodial $SU(2)_C$ symmetry even in the $g_Y = 0$ limit.

	β_1	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}	α_{11}
$SU(2)_C$	×	○	○	○	○	○	×	×	×	×	×	×
#(gauge)	2	2	3	3	4	4	4	4	2	3	4	3

- From custodial symmetry, it is expected

$$\alpha_{1,2,3,4,5} \gg \alpha_{6,7,8,9,10,11} .$$

- β_1, α_1 and α_8 contribute to the electroweak oblique correction:

$$S = -16\pi\alpha_1, \qquad \alpha T = 2\beta_1, \qquad U = -16\pi\alpha_8 .$$

They are constrained severely from the electroweak precision tests.

- α_2, α_3, α_9 and α_{11} contribute to the triple-gauge-vertices:

$$
\begin{aligned}
\mathcal{L}_{\rm TGV} \quad = \quad & -ie\frac{c_{M_Z}}{s_{M_Z}}[1+\Delta\kappa_Z]W^+_\mu W^-_\nu Z^{\mu\nu} \\
& -ie[1+\Delta\kappa_\gamma]W^+_\mu W^-_\nu A^{\mu\nu} \\
& -ie\frac{c_{M_Z}}{s_{M_Z}}[1+\Delta g_1^Z](W^{+\mu\nu}W^-_\mu - W^{-\mu\nu}W^+_\mu)Z_\nu \\
& -ie(W^{+\mu\nu}W^-_\mu - W^{-\mu\nu}W^+_\mu)A_\nu \\
& -e\frac{c_{M_Z}}{s_{M_Z}}g_5^Z\epsilon^{\mu\nu\rho\lambda}[W^+_\mu(\partial_\rho W^-_\nu) - (\partial_\rho W^+_\mu)W^-_\nu]Z_\lambda
\end{aligned}
$$

Hagiwara-Peccei-Zeppenfeld-Hikasa, NPB282 (1987) 253.

$$
\begin{aligned}
\Delta g_1^Z &= \frac{1}{c^2 - s^2}\beta_1 + \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1 + \frac{1}{s^2 c^2} e^2 \alpha_3 \\
\Delta \kappa_Z &= \frac{1}{c^2 - s^2}\beta_1 + \frac{1}{c^2(c^2 - s^2)} e^2 \alpha_1 + \frac{1}{c^2} e^2 (\alpha_1 - \alpha_2) + \frac{1}{s^2} e^2 (\alpha_3 - \alpha_8 + \alpha_9), \\
\Delta \kappa_\gamma &= \frac{1}{s^2} e^2 (-\alpha_1 + \alpha_2 + \alpha_3 - \alpha_8 + \alpha_9), \\
g_5^Z &= \frac{1}{s^2 c^2} e^2 \alpha_{11}
\end{aligned}
$$

If custodial symmetry violating term α_9 is negligible,

$$
\Delta \kappa_Z = \Delta g_1^Z - \frac{s^2}{c^2} \Delta \kappa_\gamma .
$$

Present experimental limits

$$
\Delta g_1^Z = -0.016 + 0.022 - 0.019, \qquad \Delta \kappa_\gamma = -0.027 + 0.044 - 0.045,
$$

lead to bounds on $\alpha_{2,3}$ at 10^{-2} level.

- $\alpha_{4,5,6,7,10}$ contribute to $WW \to WW$ process. Future colliders may be sensitive to these parameters:

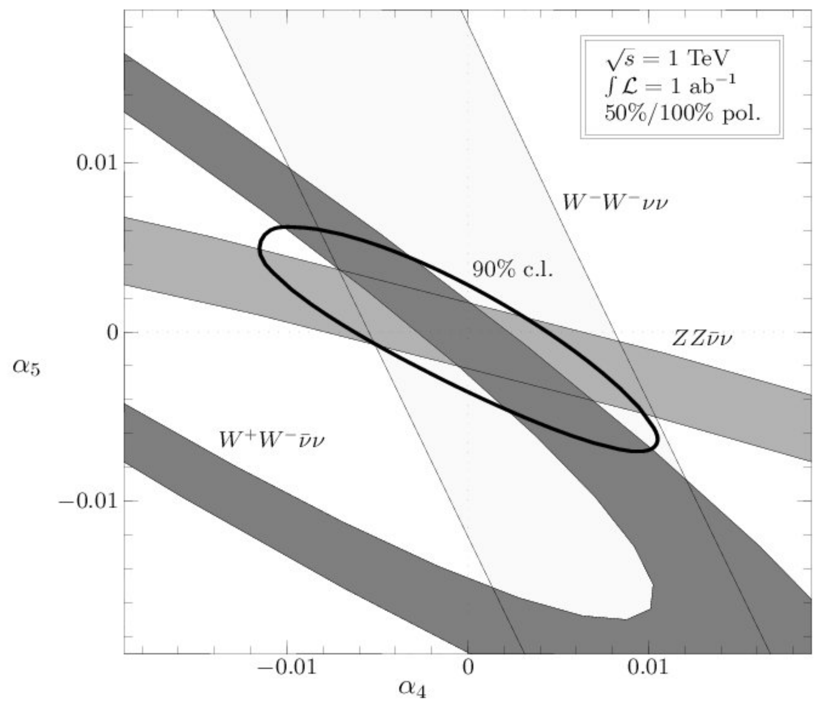


Figure 1: *Exclusion contours for the hypothesis* $\alpha_{4,5} = 0$, *assuming* $\sqrt{s} = 1$ *TeV and an integrated* e^+e^- *luminosity of* $\int \mathcal{L} = 1$ ab^{-1} *(50%/100% polarization). The 90% exclusion line has been obtained by combining the* W^+W^- *and* ZZ *channels (dark gray). The contour for the* W^-W^- *channel (light gray) corresponds to an integrated* e^-e^- *luminosity of* $\int \mathcal{L} = 100$ fb^{-1} *(100% polarization).*

E. Boos, H.-J. He et al., PRD61 (2000) 077901.

Effects of heavy fermion loop

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \qquad U_R, \quad D_R$$

Heavy U and D $\Rightarrow$ Large Yukawa coupling

violation of decoupling theorem

From S and U

$$\alpha_1 = -\frac{N}{96\pi^2}\left[1 - 2Y_{Q_L}\ln\frac{m_U^2}{m_D^2}\right],$$

$$\alpha_8 = -\frac{N}{96\pi^2}\left[-\frac{5m_U^4 - 22m_U^2 m_D^2 + 5m_D^4}{3(m_U^2 - m_D^2)^2} + \frac{m_U^6 - 3m_U^4 m_D^2 - 3m_U^2 m_D^4 + m_D^4}{(m_U^2 - m_D^2)^3}\ln\frac{m_U^2}{m_D^2}\right].$$

T.Inami, C.S. Lim, B.Takeuchi and M.T., PLB381 (1996) 458

From the TGV analysis,

$$\alpha_2 = -\frac{N}{96\pi^2}\left[1 - Y_{Q_L}\left((2+\frac{6m_U^2 m_D^2}{(m_U^2-m_D^2)^2})\ln\frac{m_U^2}{m_D^2} - 3\frac{m_U^2+m_D^2}{m_U^2-m_D^2}\right)\right],$$

$$\alpha_3 = -\frac{N}{64\pi^2}\left[\frac{m_U^2 m_D^2(m_U^2+m_D^2)}{(m_U^2-m_D^2)^3}\ln\frac{m_U^2}{m_D^2} + \frac{m_U^4 - 6m_U^2 m_D^2 + m_D^4}{2(m_U^2-m_D^2)^2}\right],$$

$$\alpha_9 = -\frac{N}{64\pi^2}\left[\frac{m_U^2 m_D^2(m_U^2+m_D^2)}{(m_U^2-m_D^2)^3}\ln\frac{m_U^2}{m_D^2} - \frac{m_U^4 + 10m_U^2 m_D^2 + m_D^4}{6(m_U^2-m_D^2)^2}\right] + \alpha_8,$$

$$\alpha_{11} = -\frac{N}{64\pi^2}\left[\frac{m_U^2 m_D^2}{(m_U^2-m_D^2)^2}\ln\frac{m_U^2}{m_D^2} - \frac{m_U^2+m_D^2}{2(m_U^2-m_D^2)}\right]$$

T. Appelquist and G.-H. Wu, PRD48 (1993) 3235.

If U and D are almost degenerated,

$$|\Delta m| \ll \hat{m}, \qquad \Delta m \equiv m_U - m_D, \qquad \hat{m} \equiv \frac{m_U + m_D}{2}$$

we find

$$\beta_1 \simeq \frac{N}{24\pi^2} \frac{(\Delta m)^2}{v^2}$$

$$\alpha_1 \simeq -\frac{N}{96\pi^2}, \quad \alpha_2 \simeq -\frac{N}{96\pi^2}, \quad \alpha_3 \simeq -\frac{N}{96\pi^2}\left[1 - \frac{(\Delta m)^2}{10\hat{m}^2}\right],$$

$$\alpha_8 \simeq -\frac{N}{96\pi^2}\frac{4(\Delta m)^2}{5\hat{m}^2}, \quad \alpha_9 \simeq -\frac{N}{96\pi^2}\frac{7(\Delta m)^2}{10\hat{m}^2}, \quad \alpha_{11} \simeq \frac{N}{96\pi^2}\frac{\Delta m}{2\hat{m}}$$

Note:

- Custodial $SU(2)_C$ symmetry in $\Delta m = 0$ limit.
- The sizes of $\alpha_{6,7,8,9,10,11}$ are extremely suppressed for $(\Delta m)^2 \ll \hat{m}^2$.
- Degenerated heavy 4th generation: $\alpha_1 = \alpha_2 = \alpha_3 \simeq 4.2 \times 10^{-3}$.

Renormalization Group

T. Appelquist and C. Bernard, PRD22 (1980) 200.

Even if we start with $\beta_1 = \alpha_i = 0$, the loop diagram of the lowest order Lagrangian $\mathcal{L}_0$ causes non-trivial running of β_1 and α_i:

$$\mu \frac{d}{d\mu} \beta_1^r(\mu) = \frac{\gamma_{\beta 1}}{(4\pi)^2}, \qquad \mu \frac{d}{d\mu} \alpha_i^r(\mu) = \frac{\gamma_{\alpha i}}{(4\pi)^2}.$$

$\gamma_{\beta 1}$	$\gamma_{\alpha 1}$	$\gamma_{\alpha 2}$	$\gamma_{\alpha 3}$	$\gamma_{\alpha 4}$	$\gamma_{\alpha 5}$
$\frac{3}{4} g_Y^2$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$-\frac{1}{6}$	$-\frac{1}{12}$

Tree level matching conditions with the Standard Model Higgs sector

$$\beta_1^r(\mu \simeq M_H) = 0, \qquad \alpha_1^r(\mu \simeq M_H) = 0$$

lead to

$$\beta^r(\mu) = -\frac{3 g_Y^2}{4(4\pi)^2} \ln \frac{M_H}{\mu}, \qquad \alpha_1^r(\mu) = -\frac{1}{6(4\pi)^2} \ln \frac{M_H}{\mu},$$

$$T \simeq \frac{8\pi}{e^2} \beta_1^r(\mu) \simeq -\frac{3}{8\pi c^2} \ln M_H, \qquad S \simeq -16\pi \alpha_1^r(\mu) \simeq \frac{1}{6\pi} \ln M_H.$$

Alternative parametrization:
(motivated by the hadron chiral perturbation theory)

Gasser and Leutwyler

$$
\begin{aligned}
\mathcal{L}_{\rm GL} \;=\;& L_1 \mathrm{tr}(D_\mu U^\dagger D^\mu U)\mathrm{tr}(D_\nu U^\dagger D^\nu U) + L_2 \mathrm{tr}(D_\mu U^\dagger D_\nu U)\mathrm{tr}(D^\mu U^\dagger D^\nu U) \\
& + i g_W L_{9L} \mathrm{tr}(D_\mu U D_\nu U^\dagger W^{\mu\nu}) + i g_Y L_{9R} \mathrm{tr}(D_\mu U^\dagger D_\nu U \frac{\tau_3}{2} B^{\mu\nu}) \\
& + g_W g_Y L_{10} \mathrm{tr}(W^{\mu\nu} U \frac{\tau_3}{2} B_{\mu\nu} U^\dagger),
\end{aligned}
$$

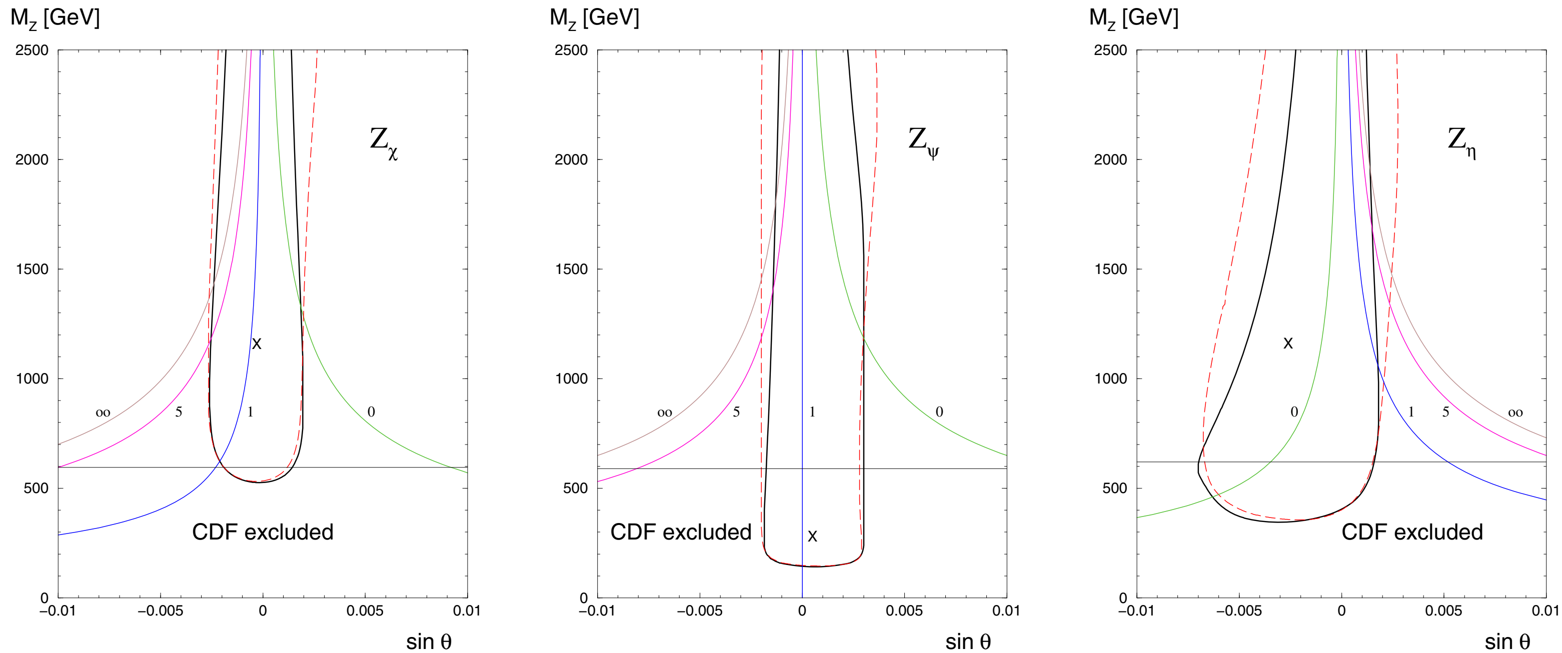
$$
\begin{aligned}
\alpha_1 &= L_{10}, \\
\alpha_2 &= -L_{9R}/2, \\
\alpha_3 &= -L_{9L}/2, \\
\alpha_4 &= L_2, \\
\alpha_5 &= L_1.
\end{aligned}
$$

§.6 "Universal" non-oblique corrections

R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, NPB703 (2004) 127.
R.S.Chivukula, E.H.Simmons, H.-J. He, M. Kurachi, and M.T., PLB603 (2004) 210.

Effects of heavy Z' cannot be fully parametrized in the S,T and U framework.

For E_6 models, dedicated study is required:



90% CL

Erler-Langacker, PLB456 (1999) 68.

Z' and W' couple with ordinary quarks/leptons only through $J_W^{a\mu}$, J_Y^{μ} in little-Higgs or Higgsless models. ("universal" models)

Fermion scattering amplitude in "universal" models: $(S, T, \alpha\delta, \Delta\rho)$

- Charged current process

$$-\mathcal{M}_{\rm CC} = \frac{(I_+ I'_- + I_- I'_+)/2}{-\left(\frac{s^2}{e^2} - \frac{S}{16\pi}\right) k^2 + \frac{1}{4\sqrt{2}G_F}\left(1 + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2}\frac{(I_+ I'_- + I_- I'_+)}{2}.$$

- Neutral current process

$$-\mathcal{M}_{\rm NC} = e^2 \frac{\mathcal{Q}\mathcal{Q}'}{-k^2} + \frac{(I_3 - s^2\mathcal{Q})(I'_3 - s^2\mathcal{Q}')}{-\left(\frac{s^2c^2}{e^2} - \frac{S}{16\pi}\right) k^2 + \frac{1}{4\sqrt{2}G_F}\left(1 - \alpha T + \frac{\alpha\delta}{4s^2c^2}\right)} + \sqrt{2}G_F \frac{\alpha\delta}{s^2c^2} I_3 I'_3 + 4\sqrt{2}G_F(\Delta\rho - \alpha T)(\mathcal{Q} - I_3)(\mathcal{Q}' - I'_3).$$

R.S.Chivukula, E.H.Simmons, H.-J. He, M. Kurachi, and M.T., PLB603 (2004) 210.

Note:

- In the absence of W' and Z' induced four-fermion couplings, S and T agree with the Peskin-Takeuchi definitions of S and T.
- $\Delta\rho$ agrees with the original ρ parameter definition:

$$\rho = 1 + \Delta\rho = \frac{G_{\rm NC}}{G_{\rm CC}},$$

  which has been measured through the low energy NC experiments.
- In the absence of $\alpha\delta$, even if $\Delta\rho \neq \alpha T$, limits on S-T is identical to the exisiting S-T limit derived from the Z-pole precision data.

An example of "universal" model

$$SU(2)_W \times U(1)_{Y1} \times U(1)_{Y2}$$

Mass matrix of neutral gauge boson

$$\frac{1}{4}(W^3_\mu \ B_{1\mu} \ B_{2\mu}) \begin{pmatrix} g_W^2 v^2 & -g_W g_{Y1} v^2 & 0 \\ -g_W g_{Y1} v^2 & g_{Y1}^2 (v^2 + v_B^2) & -g_{Y1} g_{Y2} v_B^2 \\ 0 & -g_{Y1} g_{Y2} v_B^2 & g_{Y2}^2 v_B^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B_1^\mu \\ B_2^\mu \end{pmatrix}$$

Interaction with quarks and leptons:

$$\mathcal{L}_{\text{int}} = g_W J_W^{a\mu} W_\mu^a + g_{Y1} J_Y^\mu B_{1\mu}.$$

For $g_W, g_{Y1} \ll g_{Y2}$, we find

$$\alpha S = -4 s^2 c^2 \frac{M_W^2}{M_{Z'}^2} \frac{g_{Y1}^2}{g_{Y2}^2}, \quad \alpha T = -s^2 \frac{M_W^2}{M_{Z'}^2} \frac{g_{Y1}^2}{g_{Y2}^2}, \quad \alpha \delta = 0, \quad \Delta \rho = 0.$$

Negative S ! Negative T !

Technicolor plus $SU(2)_W \times U(1)_{Y1} \times U(1)_{Y2}$

- Technicolor large positive S may be canceled by the negative S from Z'.
- Negative T from extra Z' may be canceled by custodial symmetry breaking in the TC sector.

Sounds very nice!

This model predicts

$$\Delta\rho - \alpha T \sim 0.3 \times 10^{-2}.$$

Alternative parametrization

R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, NPB703 (2004) 127.

Dimension 6 operators

$$\mathcal{L} = c_{WB}\mathcal{O}_{WB} + c_H\mathcal{O}_H + c_{WW}\mathcal{O}_{WW} + c_{BB}\mathcal{O}_{BB},$$

with

$$\mathcal{O}_{WB} = \frac{1}{g_W g_Y}(\phi^\dagger \tau^a \phi) W^a_{\mu\nu} B^{\mu\nu}, \qquad \mathcal{O}_H = |\phi^\dagger D_\mu \phi|^2,$$

$$\mathcal{O}_{WW} = \frac{1}{2g_W^2}(D_\rho W^a_{\mu\nu})^2, \qquad \mathcal{O}_{BB} = \frac{1}{2g_Y^2}(\partial_\rho B_{\mu\nu})^2.$$

$$\hat{S} = \frac{c}{s} v^2 c_{WB}, \quad \hat{T} = -\frac{v^2}{2} c_H, \quad W = -\frac{g_W^2 v^2}{2} c_{WW}, \quad Y = -\frac{g_Y^2 v^2}{2} c_{BB}.$$

Dictionary between $(S, T, \alpha\delta, \Delta\rho)$ and $(\hat{S}, \hat{T}, W, Y)$

$$\begin{aligned} \alpha S &= 4s^2(\hat{S} - Y - W), \\ \alpha T &= \hat{T} - \frac{s^2}{c^2} Y, \\ \alpha\delta &= 4s^2 c^2 W, \\ \Delta\rho &= \hat{T}. \end{aligned}$$

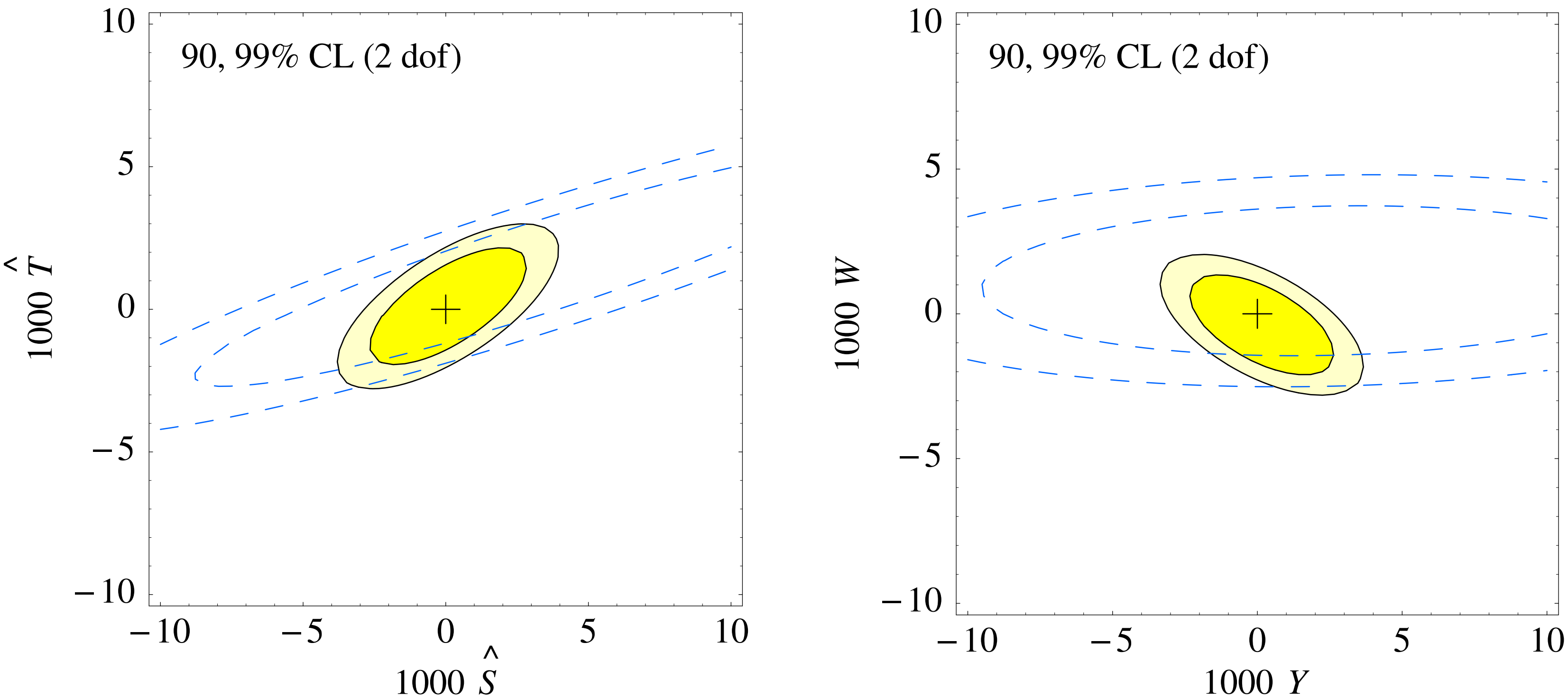
$$\begin{aligned} \hat{S} &= \frac{1}{4s^2}\left(\alpha S + 4c^2(\Delta\rho - \alpha T) + \frac{\alpha\delta}{c^2}\right), \\ \hat{T} &= \Delta\rho, \\ W &= \frac{\alpha\delta}{4s^2c^2}, \\ Y &= \frac{c^2}{s^2}(\Delta\rho - \alpha T). \end{aligned}$$

- In the absence of "universal" non-oblique corrections, $W = Y = 0$ and

$$\alpha S = 4s^2 \hat{S}, \qquad \alpha T = \hat{T}.$$

- Technicolor plus $SU(2)_W \times U(1)_{Y1} \times U(1)_{Y2}$ model predicts $\hat{S} \simeq 0$, $\hat{T} \simeq 0$, $W \simeq 0$, but $Y \simeq 10^{-2}$.

$(\hat{S}, \hat{T}, W, Y)$-fit with and without LEP2 data of $\sigma(e^+e^- \to f\bar{f})$



$(\hat{S}, \hat{T})$ for generic (W, Y) $\qquad$ (W, Y) for generic $(\hat{S}, \hat{T})$

Barbieri et al. hep-ph/0405042

§.7 まとめ

- New physics scenarios which affect EW physics through *non-decoupling* radiative corrections are severely constrained by the precision data:
  - Heavy 4th generation
  - Technicolor models
- The precision data suggest that, in the standard model framework, Higgs should be light $M_H \leq 199$GeV. Heavier Higgs thus indicates BSM.
- Electroweak chiral perturbation theory provides a powerful tool to catalogue all possible *non-decoupling* corrections:
  S-T-U, TGV, $WW \to WW$
- In the "universal" models (little-Higgs, Higgsless, etc.), S-T fit is not enough. We can do better by using $(S, T, \alpha\delta, \Delta\rho)$ or $(\hat{S}, \hat{T}, W, Y)$.
  $\Rightarrow$ Prof. Chivukula's lecture for Higgsless models.

References

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