ヒッグスレス模型と
電弱およびフレーバー精密測定

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Higgs particle is a hypothetical particle introduced so as to explain the origin of mass in the particle physics.

Experimentalists have not yet found the Higgs, however. Moreover, Higgs sector of the standard model is known to be problematic.

Is it possible to construct models without a Higgs, then?

The role of the Higgs boson in the SM:

- Renormalizability :

  W and Z are gauge bosons (universality of weak interaction).

  Explicit breaking of electroweak gauge symmetry makes the theory *non-renormalizable*. We need, at least, one Higgs boson so as to feed W and Z masses through *spontaneous breaking*.

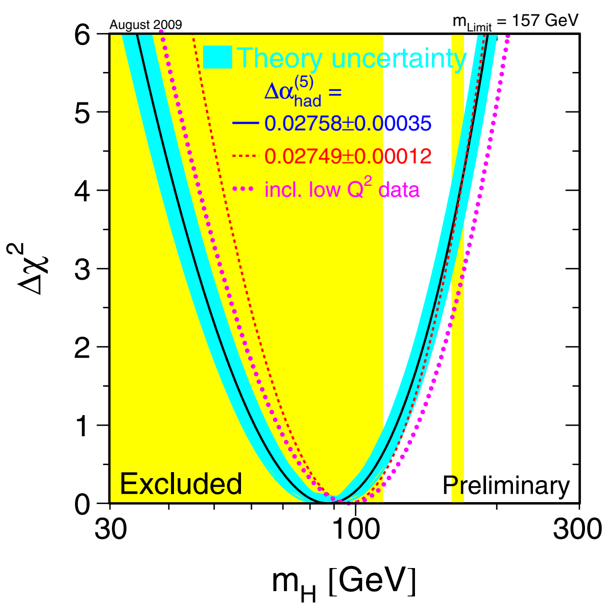
- Unitarity :

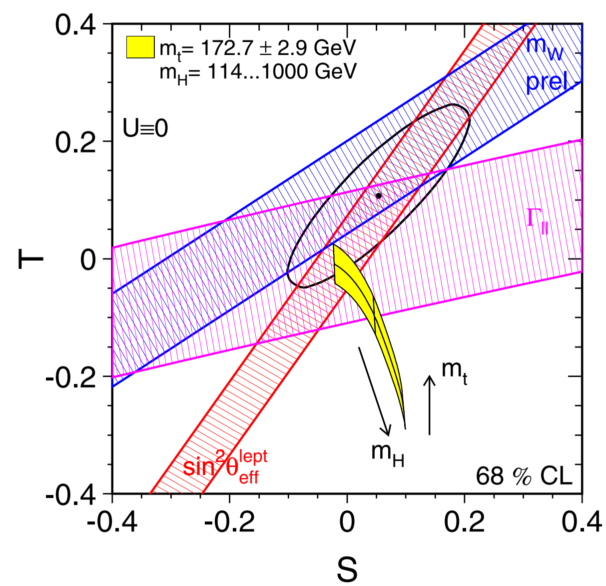
  The longitudinal W boson (W_L) scattering amplitude grows as the CM energy increases. If there is no Higgs boson, it eventually violates the unitarity.

Life without a Higgs

Renormalizability :

New physics (cutoff scale of SM) is believed to exist at TeV. In principle, renormalizability is not a primary issue in this sense. However, the lack of renormalizability usually implies a loss of robust predictability. How can we ensure the consistency with the existing precision electroweak measurements without introducing a Higgs boson then?



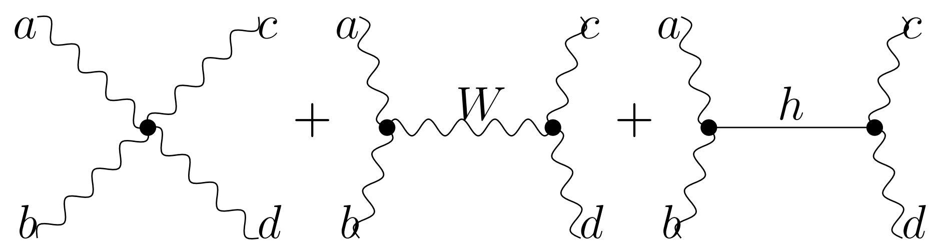


Unitarity:

B.W.Lee, C.Quigg, and H.B.Thacker

In the standard model, a Higgs boson (scalar resonance) *"unitarizes"* the $W_L W_L$ scattering amplitude:

$$i\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) =$$



$+$crossed.

For $E \gg M_W$

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = \frac{s}{v^2}\frac{M_H^2}{M_H^2 - s}\delta^{ab}\delta^{cd} + \frac{t}{v^2}\frac{M_H^2}{M_H^2 - t}\delta^{ac}\delta^{bd} + \frac{u}{v^2}\frac{M_H^2}{M_H^2 - u}\delta^{ad}\delta^{bc},$$

with

$$M_H^2 = \lambda v^2, \qquad v \simeq 250 \text{ GeV}.$$

- $W_L W_L$ scattering amplitude remains perturbative even at high energy scale $\sqrt{s} \gg 1$ TeV thanks to the light Higgs exchange.

How can we ensure the $W_L W_L$ scattering unitarity (or calculability) without introducing a Higgs boson?

Unitarity in the WW Scattering

Unitarity in the $W_L W_L$ scatteing

Let us consider the longitudinally polarized W (W_L). It's polarization vector

$$\epsilon^{\mu}_{(L)} = \frac{E}{M_W} \begin{pmatrix} \frac{|\vec{p}|}{E} \\ \frac{\vec{p}}{|\vec{p}|} \end{pmatrix}, \qquad E^2 = |\vec{p}|^2 + M_W^2, \quad \epsilon_{(L)\mu}\epsilon^{\mu}_{(L)} = -1$$

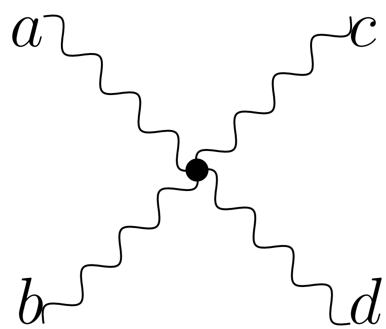
grows for large $E \gg M_W$. Naive power counting in E suggests

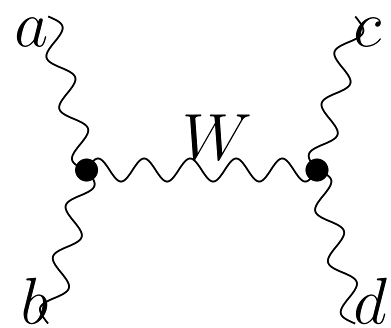
$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) \propto |\epsilon_{(L)\mu}|^4 \sim \frac{E^4}{M_W^4}$$

Unitarity seems to be violated in the high energy $W_L W_L$ scattering.

For simplicity, we consider the $g_Y = 0$ case. $(Z = W^3)$

Two Feynman diagrams contributing to the WW scattering

$i\mathcal{M}_{\text{gauge}}(ab \to cd) =$ 

$+$ 

$+$ crossed.

Contribution from the $4W$ vertex (terms proportional to $\delta^{ab}\delta^{cd}$)

$$g_{WWWW}\frac{E^4}{M_W^4}\left\{-(1+\cos\theta)[3-\cos\theta-2\frac{M_W^2}{E^2}]\right.$$
$$\left.-(1-\cos\theta)[3+\cos\theta-2\frac{M_W^2}{E^2}]\right\}$$

t-channel W exchange (terms proportional to $\delta^{ab}\delta^{cd}$)

$$g_{WWW}^2 \frac{E^4}{M_W^4}\left\{(1-\cos\theta)[3+\cos\theta-2\frac{M_W^2}{E^2}]+\frac{1}{2}(1+11\cos\theta)\frac{M_W^2}{E^2}\right\}+\cdots$$

u-channel W exchange (terms proportional to $\delta^{ab}\delta^{cd}$)

$$g_{WWW}^2 \frac{E^4}{M_W^4}\left\{(1+\cos\theta)[3-\cos\theta-2\frac{M_W^2}{E^2}]+\frac{1}{2}(1-11\cos\theta)\frac{M_W^2}{E^2}\right\}+\cdots$$

Each diagram behaves $\sim E^4/M_W^4$ at high energy.
The leading E^4/M_W^4 term cancels in the amplitude thanks to the Ward-Takahashi identity

$$g_{WWWW} = g_{WWW}^2 = g_W^2$$

of the gauge symmetry.

$$\mathcal{M}_{\rm gauge}(ab \to cd) = \delta^{ab}\delta^{cd} g_W^2 \frac{E^4}{M_W^4}\frac{M_W^2}{E^2} + \cdots = \delta^{ab}\delta^{cd} g_W^2 \frac{E^2}{M_W^2} + \cdots$$

We use $E^2 = s/4$, $M_W^2 = g_W^2 v^2/4$

$$\mathcal{M}_{\rm gauge}(ab \to cd) = \frac{s}{v^2}\delta^{ab}\delta^{cd} + \frac{t}{v^2}\delta^{ac}\delta^{bd} + \frac{u}{v^2}\delta^{ad}\delta^{bc} + \cdots.$$

This form agrees with the low energy theorem (equivalence theorem)

B. W. Lee, C. Quigg and H. B. Thacker, Phys. Rev. Lett. **38**, 883 (1977); Phys. Rev. D **16**, 1519 (1977).

This is clear because

NG boson $\Rightarrow W_L$

(Higgs mechanism)

Unitarity

If the $W_L W_L$ scattering amplitude is completely given by the low energy theorem

The probability of the $W_L W_L$ scattering exceeds unity at the $s = 8\pi v^2$ eneryg scale.

$\Downarrow$

unitarity violation

Two possibilities

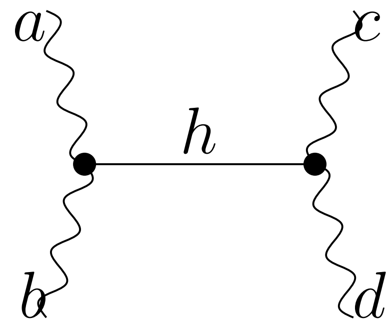
Unitarity bound : $\sqrt{8\pi}v \simeq 1.2$TeV

- perturbative case
  The $W_L W_L$ scattering behavior is modified thanks to the existence of particles lighter than the unitarity bound (predictable model.)
- non-perturbative casse
  The theory beoomes non-perturbative above the unitarity bound. The unitarity should be recovered in a non-perturbative manner. (predictability may be lost.)

Perturbative unitarity in the standard model Higgs sector

Higgs exchange diagram

$$i\mathcal{M}_{\text{Higgs}}(ab \to cd) =$$



$+$ crossed.

$$\mathcal{M}_{\text{Higgs}}(ab \to cd) = g_{hWW}^2 \frac{s^2}{M_W^4} \frac{1}{M_h^2 - s} \delta^{ab} \delta^{cd} + \cdots .$$

Using the standard model Higgs relation

$$g_{hWW} = \frac{M_W^2}{v}$$

we notice that the $s \sim E^2$ term cancels

$$\mathcal{M}(ab \to cd) = \mathcal{M}_{\text{gauge}} + \mathcal{M}_{\text{Higgs}} = \frac{s}{v^2} \frac{M_h^2}{M_h^2 - s} \delta^{ab} \delta^{cd} + \cdots .$$

- The amplitude agrees with the low energy theorem at $s \ll M_h^2 = \lambda v^2$.
- The amplitude approaches to a constant λ at the region $s \gg M_h^2 = \lambda v^2$. The theory is perturbative if the constant λ is sufficiently small.

It is easy to extend this to multi-Higgs models

From the perturbative unitarity requirement of the $W_L W_L$ scattering

$$4 \sum_n g^2_{h_{(n)} WW} = (4 g_{WWWW} - 3 g^2_{WWW}) M_W^2$$

hWW coupling $\Leftrightarrow$ perturbative unitarity $\Leftrightarrow$ Essence of a "Higgs"

Physics ensuring the unitarity in the $W_L W_L$ scattering

- $\mathcal{O}(E^4)$ cancellation

$$g_{WWWW} = g_{WWW}^2$$

gauge symmetry

- $\mathcal{O}(E^2)$ cancellation

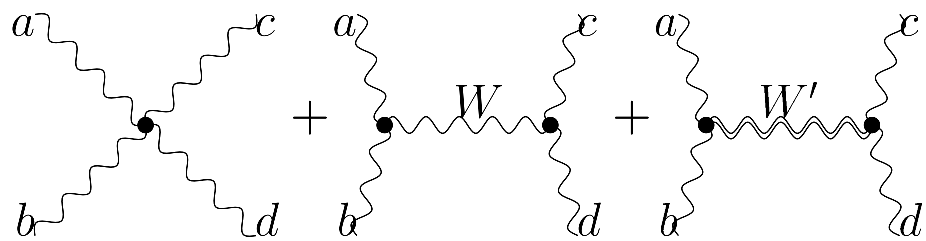
$$4\sum_n g_{h_{(n)}WW}^2 = (4g_{WWWW} - 3g_{WWW}^2)M_W^2$$

unitarity sum rules

The $h_{(n)}WW$ coupling determimes the Higgs property of $h_{(n)}$.

Can a spin-1 resonance unitarize the $W_L W_L$ scattering amplitude?

$i\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) =$



+crossed.

Answer: *Yes!* *if we suitably adjust WWW' coupling.*

$$\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = \frac{1}{3v^2}\left((s-u)\frac{M_{W'}^2}{M_{W'}^2 - t} + (s-t)\frac{M_{W'}^2}{M_{W'}^2 - u}\right)\delta^{ab}\delta^{cd} + \cdots$$

Cancellation of bad high-energy behavior is achieved through *exchange of massive spin-1 particle W'*.

Note, however,

we need to introduce yet another massive vector particle W'' so as to unitarize the $W'_L W'_L \to W'_L W'_L$ amplitude ....

$$\Downarrow$$

A tower of massive vector particles:

$$W, \quad W', \quad W'', \quad W''', \cdots$$

This situation is naturally realized in gauge theory with an *extra dimension*

A tower of massive Kaluza-Klein modes

Chivukula, Dicus and He ; Csaki, Grojean, Murayama, Pilo and Terning

Gauge symmetry breaking through boundary conditions

Unitarity sum rules

- Spin-0 (Higgs) exchange case:

$$g_{WWWW} = g_{WWW}^2, \qquad g_{WWWW} M_W^2 = 4 \sum_n g_{h_{(n)}WW}^2$$

- Spin-1 exchange case (Higgsless):

$$g_{WWWW} = \sum_n g_{WWW_{(n)}}^2, \qquad 4 g_{WWWW} M_W^2 = 3 \sum_n g_{WWW_{(n)}}^2 M_{W_{(n)}}^2$$

- If there exist spin-0 and spin-1 simultaneously (gaugephobic Higgs): Cacciapaglia et al. `hep-ph/0611358`, `c.f. Hikasa and Igi`

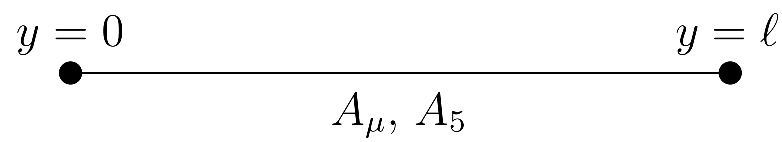
$$g_{WWWW} = \sum_n g_{WWW_{(n)}}^2,$$

$$4 g_{WWWW} M_W^2 = 3 \sum_n g_{WWW_{(n)}}^2 M_{W_{(n)}}^2 + 4 \sum_n g_{h_{(n)}WW}^2$$

Higgsless models in 5D

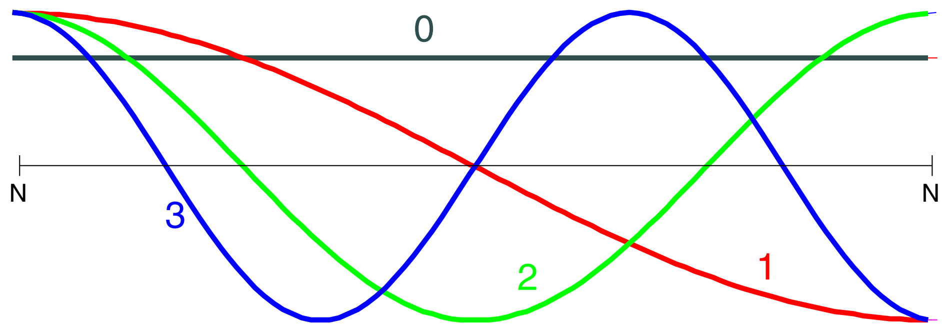
Gauge symmetry breaking through boundary conditions

5D gauge theory with an interval extra dimension



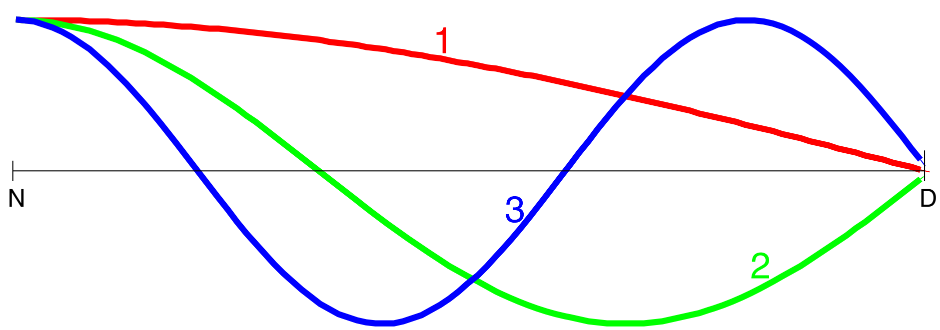
BC: Neumann(N)? Dirichlet(D)?

1. $\partial_y A_\mu(x, y)|_{y=0} = 0$ (N), $\quad \partial_y A_\mu(x, y)|_{y=\ell} = 0$ (N) [NN]



massless spin-1 field: unbroken 4D gauge symmetry

2. $\partial_y A_\mu(x, y)|_{y=0} = 0$ (N), $\quad A_\mu(x, y)|_{y=\ell} = 0$ (D) [ND]



absence of massless spi-1 field: 4D gauge sym is broken

4D gauge sym and spectrum

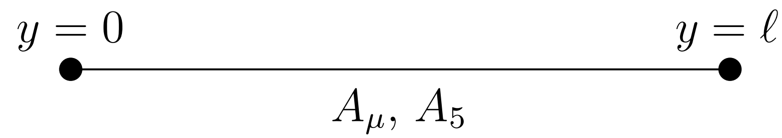
In addition to the massive spin-1 KK particles, we have

1. [NN]: massless spin-1 (unbroken gauge sym)
   photon
2. [ND]: absence of massless particle (broken gauge sym)
3. [DN]: absence of massless particle (broken gauge sym)
   $W^{\pm}$, Z
4. [DD]: massless spin-0 (gauge and global syms are broken)

Applying this mechanism to EWSB, we can push up the unitarity vilation scale around 10TeV.
(little Hierarchy)

- R. Sekhar Chivukula, D. A. Dicus and H. J. He, "Unitarity of compactified five dimensional Yang-Mills theory," Phys. Lett. B **525**, 175 (2002) [arXiv:hep-ph/0111016].
- C. Csaki, C. Grojean, H. Murayama, L. Pilo and J. Terning, "Gauge theories on an interval: Unitarity without a Higgs," Phys. Rev. D **69**, 055006 (2004) [arXiv:hep-ph/0305237].

Brane localized Higgs field (aka RS model)



BC: Neumann BC at both brane

$\partial_y A_\mu(x,y)|_{y=0} = 0$ (N), $\quad \partial_y A_\mu(x,y)|_{y=\ell} = 0$ (N) [NN]

Brane localized Higgs ϕ:

$$S_{\rm Higgs} = \int dy \delta(y - \ell + \epsilon) \left[(D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) \right]$$

$\Downarrow$ (VEV v_b)

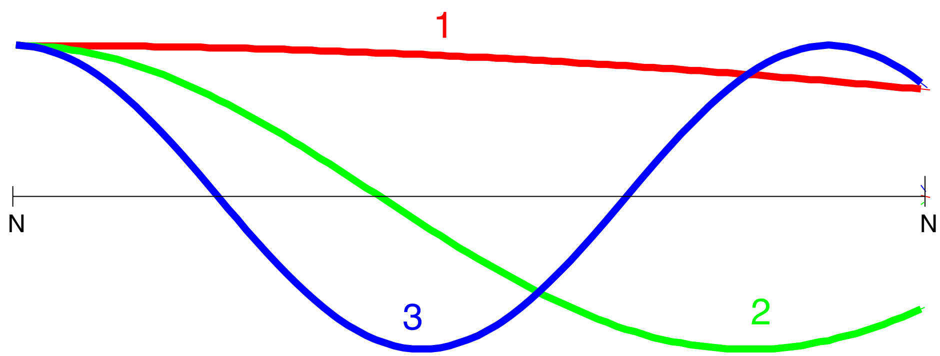
$$\int dy \delta(y - \ell + \epsilon) v_b^2 A_\mu A^\mu$$

KK mode equation for the gauge field

$$\left[-\partial_y^2 + \delta(y - \ell + \epsilon) g^2 v_b^2 \right] \chi^{(n)}(y) = M_n^2 \chi^{(n)}(y)$$

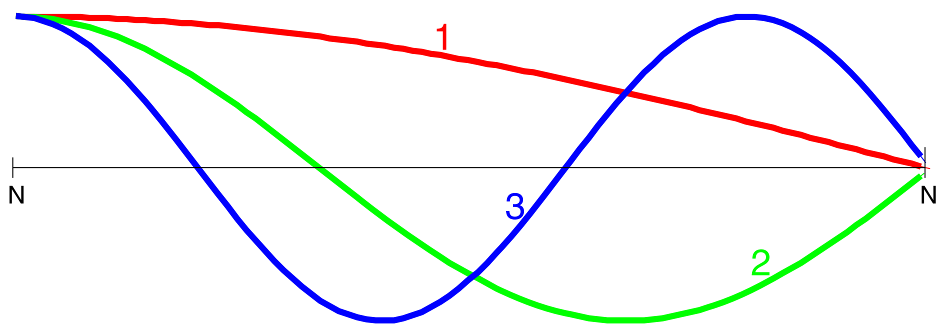
$\Leftrightarrow$ 1-dim Schroedinger eq. with δ-function repulsive force

1. finite v_b case



δ-function repulsive force at $y = \ell$ brane affects the wave-function form.

2. $v_b \longrightarrow \infty$ case



δ-function repulsive force at $y = \ell$ brane affects the effective boundary condition at the brane.

Remarks

- Brane localized Higgs with an infinite VEV.

$$\Updownarrow \text{ (equivalent)}$$

Dirichlet BC (Higgsless)

- The KK gauge boson spectrum remains finite around the compactification scale even in the infinite Higgs VEV limit.

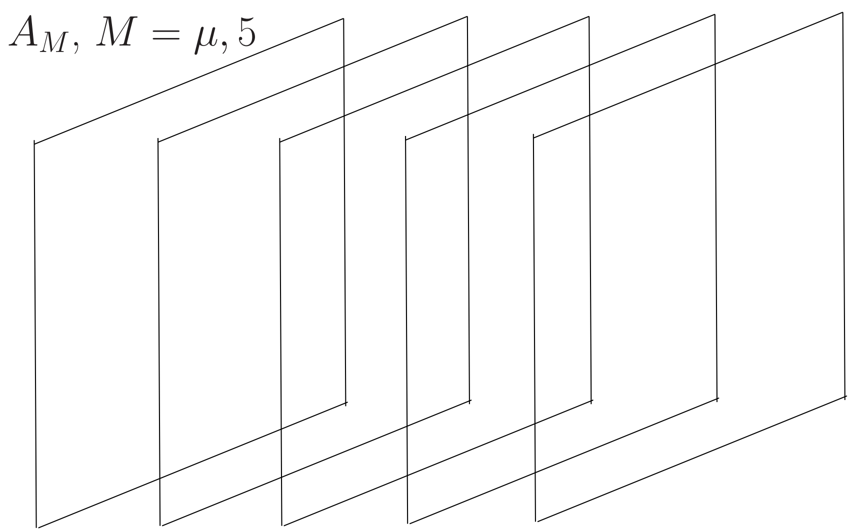
- Higgsless models can be regarded as a variant of usual RS model.

Effective theory viewpoint
— Deconstruction —

Deconstruction of boundary conditions

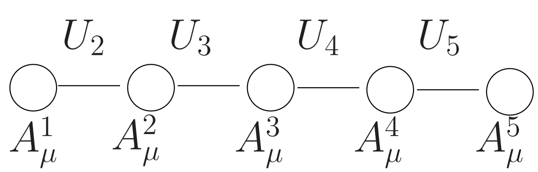
Deconstruction (latticization) of extra dimension

Arkani-Hamed, Cohen and Georgi ; Hill, Pokorski and Wang



a : lattice spacing

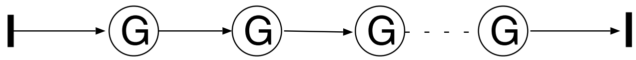
- $A_\mu^j = A_\mu(x, y = ja)$: gauge field at site j
- $U_j = \exp(i \int_{(j-1)a}^{ja} dy A_5(x,y))$: link field. non-linear σ model field.

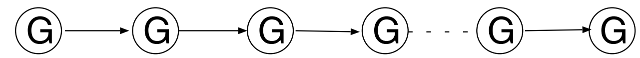


moose diagram

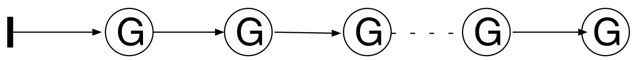
Note: Moose model can be viewed as a generalization of Bando-Kugo-Yamawaki's Hidden Local Symmetry (HLS) model (Phys.Rep.164,217(1988)) + Georgi's vector symmetry model (NPB331,311(1990)).

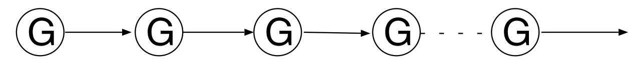
Deconstructions of an interval in "moose" notation:

[DD] 

[NN] 

$$\#(U_j) = \#(A^j_\mu) + 1. \qquad \#(U_j) = \#(A^j_\mu) - 1.$$

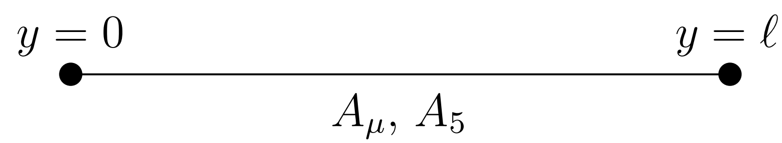
[DN] 

[ND] 

$$\#(U_j) = \#(A^j_\mu). \qquad \#(U_j) = \#(A^j_\mu).$$

which correspond to 5D gauge theories with an interval compactification:

H.-J. He, hep-ph/0412113



[DD] $A_\mu(x,y)|_{y=0} = 0$ (D), $\partial_5 A_5(x,y)|_{y=0} = 0$ (N),
$A_\mu(x,y)|_{y=\ell} = 0$ (D), $\partial_5 A_5(x,y)|_{y=\ell} = 0$ (N).

[NN] $\partial_5 A_\mu(x,y)|_{y=0} = 0$ (N), $A_5(x,y)|_{y=0} = 0$ (D),
$\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N), $A_5(x,y)|_{y=\ell} = 0$ (D).

[DN] $A_\mu(x,y)|_{y=0} = 0$ (D), $\partial_5 A_5(x,y)|_{y=0} = 0$ (N),
$\partial_5 A_\mu(x,y)|_{y=\ell} = 0$ (N), $A_5(x,y)|_{y=\ell} = 0$ (D).

Spectrum and 4D gauge symmetry:

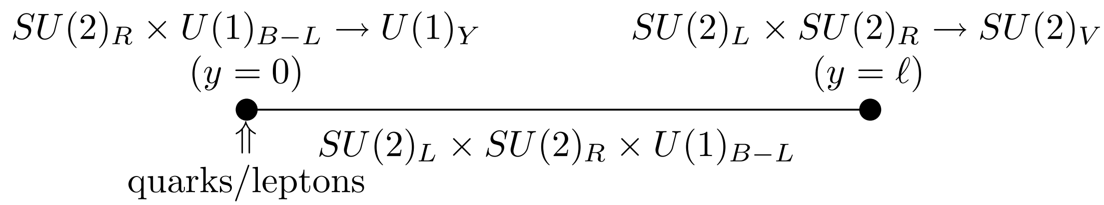
In addition to a tower of massive spin-1 KK-modes, we have

1. [DD]: massless spin-0 particle. 4D gauge sym. is all broken.
2. [NN]: massless spin-1 particle. unbroken 4D gauge sym.
3. [DN]: no massless particle. 4D gauge sym. is all broken.
4. [ND]: no massless particle. 4D gauge sym. is all broken.

Advantages for deconstruction in 5D Higgsless models

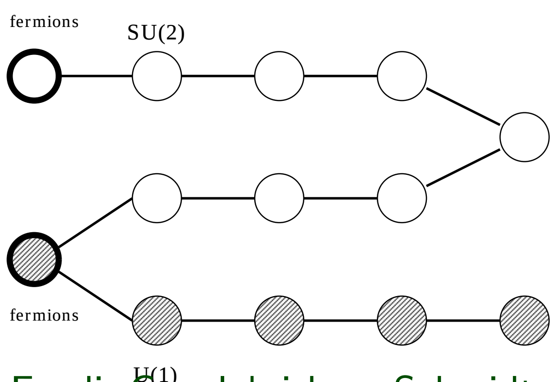
- Familiar language of spontaneous gauge symmetry breaking (gauged nonlinear σ model).
- Easier to understand the physics behind the delay of unitarity violation.
- Easier to calculate corrections to electroweak interactions.
- Allowing for arbitrary background 5D geometry, spatially dependent gauge couplings, and brane kinetic terms.
- Easier to perform loop analysis using well-known chiral perturbation method.

5D Higgsless model



Csaki et al. PRL92 (2004) 101802; Nomura JHEP 11 (2003) 050; Barbieri et al. PLB591 (2004) 141.

and its deconstruction



Foadi, Gopalakrishna, Schmidt ; Casalbuoni, De Curtis and Dominici ; Chivukula, Simmons, He, Kurachi and M.T.; Georgi

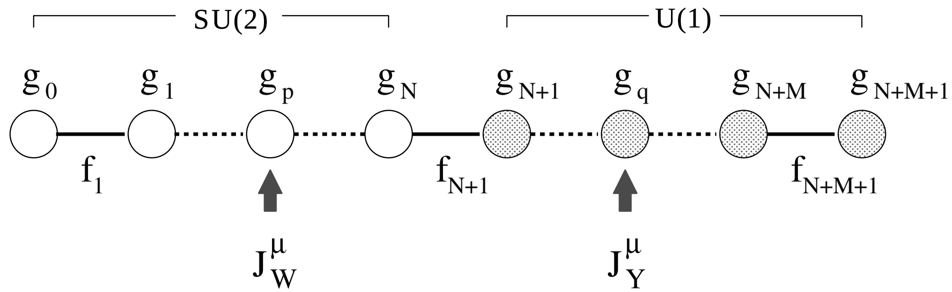
5D Higgsless models are described by linear moose.

c.f. BESS model (Casalbuoni et al., PLB 155 95 (1985))

$\Leftrightarrow$ HLS model (Bando-Kugo-Yamawaki) & Vector limit model (Georgi)

Linear moose models

General linear moose with localized J_W^μ and J_Y^μ:



$$\mathcal{L} = \frac{1}{4}\sum_{j=1}^{N+M+1} f_j^2 \text{tr}((D_\mu U_j)^\dagger (D^\mu U_j)) - \sum_{j=0}^{N+M+1} \frac{1}{4g_j^2} F_{\mu\nu}^j F^{j\mu\nu}$$

with
$$D_\mu U_j = \partial_\mu U_j - iA_\mu^{j-1} U_j + iU_j A_\mu^j .$$

Low energy Fermi-constant: (EW symmetry is broken collectively)

$$\sqrt{2} G_F = \frac{1}{v_{\rm CC}^2} = \sum_{j=p+1}^{N+1} \frac{1}{f_j^2}, \qquad \frac{1}{v_{\rm NC}^2} = \sum_{j=p+1}^{q} \frac{1}{f_j^2}.$$

Low energy QED coupling:
$$\frac{1}{e^2} = \sum_{j=0}^{N+M+1} \frac{1}{g_j^2}.$$

c.f. HLS model (Bando-Kugo-Yamawaki)

- Delay of unitarity violation

  For large $N-p$, we are able to achieve delay of unitarity violation

$$\min(f_j) \gg v \simeq 250 \text{ GeV}, \qquad \sqrt{8\pi}\min(f_j) \gg \sqrt{8\pi}v \simeq 1.2\text{TeV}.$$

  Note, however,

$$N \lesssim 100$$

  if we assume perturbative gauge coupling : $g_j^2/(4\pi) \lesssim 1$.

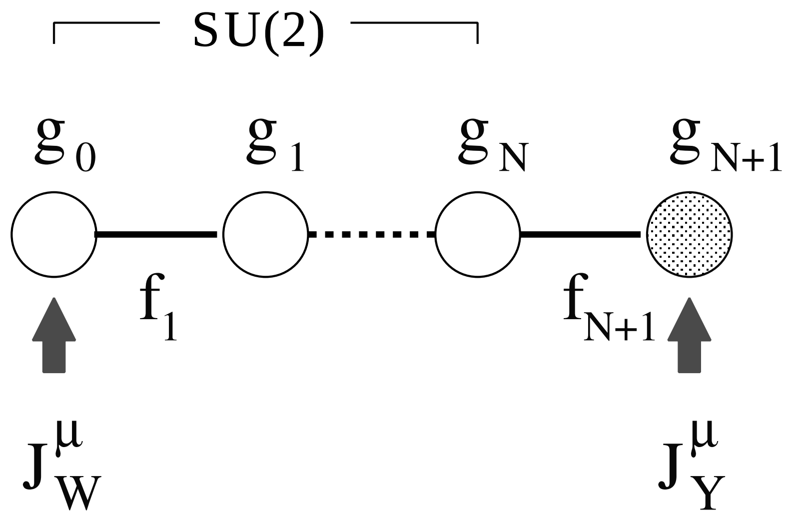
  The perturbative unitarity is eventually violated at or below 10 TeV.

- ρ parameter

$$\rho = 1 + \Delta\rho \equiv \frac{v_{CC}^2}{v_{NC}^2} = 1 + v^2 \sum_{j=N+2}^{q} \frac{1}{f_j^2} \geq 1.$$

  In this talk, we concentrate our attention on "Case I" models ($q = N+1$) in which $\Delta\rho = 0$ is satisfied automatically.

The model we consider in this talk



General Sum Rules

Chivukula, He, Kurachi, Simmons, M.T.

Phys.Rev.D78, 095003 (2008) [arXiv:0808.1682 [hep-ph]]

General Sum Rules

We try to genelarize the unitarity sum rule

$$g_{WWWW} = \sum_n g^2_{WWW_{(n)}}, \qquad 4 g_{WWWW} M_W^2 = 3 \sum_n g^2_{WWW_{(n)}} M^2_{W_{(n)}}$$

- Finite deconstruction
- Inelastic scattering

$$nn \to mm$$

- Generalize to the transverse gauge boson scattering such as $LL \to LT$, $LL \to TT$, $LT \to TT$.

$\Rightarrow$ important for the collider confirmation of the Higgsless scenario

We deduce the sum rules using the equivalence theorems

$$M(L_{(n)}L_{(n)} \to L_{(m)}L_{(m)}) \simeq M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}\pi_{(m)}), \qquad E^4, E^2, (E^0)$$

$$M(L_{(n)}L_{(n)} \to L_{(m)}T_{(m)}) \simeq M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}T_{(m)}), \qquad E^3, E^1$$

$$M(L_{(n)}L_{(n)} \to T_{(m)}T_{(m)}) \simeq M(\pi_{(n)}\pi_{(n)} \to T_{(m)}T_{(m)}), \qquad E^2, E^0$$

$$M(L_{(n)}T_{(n)} \to T_{(m)}T_{(m)}) \simeq M(\pi_{(n)}T_{(n)} \to T_{(m)}T_{(m)}), \qquad E^1$$

In the continuum limit ($N \to \infty$),

$$M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}\pi_{(m)}) = 0, \qquad M(\pi_{(n)}\pi_{(n)} \to \pi_{(m)}T_{(m)}) = 0$$

Results

$$G_4^{nnmm} = \sum_k G_3^{nnk} G_3^{mmk} = \sum_k G_3^{nmk} G_3^{nmk},$$

$$2(M_n^2 + M_m^2) G_4^{nnmm} + \sum_k (G_3^{nmk})^2 \left[\frac{(M_n^2 - M_m^2)^2}{M_k^2} - 3M_k^2 \right]$$

$$= \frac{4}{v^2} M_n^2 M_m^2 \tilde{G}_4^{nnmm},$$

$$\vdots$$

Here

$$G_4^{nm\ell k} \equiv g_{W_{(n)} W_{(m)} W_{(\ell)} W_{(k)}}, \quad G_3^{nm\ell} \equiv g_{W_{(n)} W_{(m)} W_{(\ell)}},$$

$$\tilde{G}_4^{nm\ell k} \equiv g_{\pi_{(n)} \pi_{(m)} \pi_{(\ell)} \pi_{(k)}},$$

In the continuum limit, we have $\tilde{G}_4^{nm\ell k} = 0$

These sum rules agree with the unitarity sum rule in the continuum limit

$$G_4^{nnnn} = \sum_k G_3^{nnk} G_3^{nnk},$$

$$4M_n^2 G_4^{nnnn} = 3 \sum_k M_k^2 (G_3^{nnk})^2,$$

Sum rules in the deconstructed Higgsless models

W mass matrix M_W^2, π mass matrix $\tilde{M}_W^2$ are given by ($g_Y = 0$),

$$M_W^2 = Q^T Q, \qquad \tilde{M}_W^2 = Q Q^T,$$

where

$$Q \equiv \frac{1}{2} \begin{pmatrix} g_0 f_1 & -g_1 f_1 & & & \\ & g_1 f_2 & -g_2 f_2 & & \\ & & \ddots & \ddots & \\ & & & g_{N-1} f_N & -g_N f_N \\ & & & & g_N f_{N+1} \end{pmatrix}.$$

Diagonalization

$$\begin{pmatrix} M_1 & & & \\ & M_2 & & \\ & & \ddots & \\ & & & M_{N+1} \end{pmatrix} = M_W^{\rm diag} = R^T Q^T \tilde{R} = \tilde{R}^T Q R$$

The matrices R, $\tilde{R}$ represent the KK gauge bson $W_{(n)}$ (NG boson $\pi_{(n)}$) "wave function" in the extra dimension.

G_3^{nmk}, $G_4^{nm\ell k}$ are given by the KK gauge boson wave function,

$$G_3^{nmk} = \sum_j g_j R_{j,n} R_{j,m} R_{j,k}, \quad G_4^{nm\ell k} = \sum_j g_j^2 R_{j,n} R_{j,m} R_{\ell,k} R_{j,k},$$

while $\tilde{G}_4^{nm\ell k}$ given by the NG boson wave function,

$$G_4^{nm\ell k} = \sum_j \frac{v^2}{f_j^2} \tilde{R}_{j,n} \tilde{R}_{j,m} \tilde{R}_{\ell,k} \tilde{R}_{j,k}.$$

A relation between $W_{(n)}$ and $\pi_{(n)}$ functions (WT identity)

$$\tilde{R} M_W^{\mathrm{diag}} = QR$$

"Completeness" of the wave function

$$\delta_{j,j'} = (RR^T)_{j,j'} = \sum_k R_{j,k} R_{j',k},$$

$$\delta_{j,j'} = (\tilde{R}\tilde{R}^T)_{j,j'} = \sum_k \tilde{R}_{j,k} \tilde{R}_{j',k}.$$

General sum rules

$\Updownarrow$

WT identities $+$ completeness relations

Example:

Derivation of

$$\sum_i G_3^{nmi} G_3^{\ell ki} = G_4^{nm\ell k}$$

$$\begin{aligned}
\sum_i G_3^{nmi} G_3^{\ell ki} &= \sum_i \sum_{j,j'} g_j g_{j'} R_{j,n} R_{j,m} R_{j,i} R_{j',n} R_{j',m} R_{j',i} \\
&= \sum_{j,j'} g_j g_{j'} R_{j,n} R_{j,m} R_{j',n} R_{j',m} \delta_{j,j'} \\
&= \sum_j g_j^2 R_{j,n} R_{j,m} R_{j,n} R_{j,m} \\
&= G_4^{nm\ell k}
\end{aligned}$$

Summary of the WT identities and the completeness relations

$$G_4^{nm\ell k} = \sum_i G_3^{nmi} G_3^{\ell ki},$$

$$\begin{aligned} \frac{4}{v^2}\tilde{G}_4^{nm\ell k} &= G_4^{nm\ell k}(M_n^2 + M_m^2 + M_\ell^2 + M_k^2) \\ &\quad - \sum_i (G_3^{nmi}G_3^{\ell ki} + G_3^{n\ell i}G_3^{mki} + G_3^{nki}G_3^{m\ell i})M_i^2, \end{aligned}$$

$$(M_n^2 - M_m^2)(M_\ell^2 - M_k^2)\sum_i \frac{G_3^{nmi}G_3^{\ell ki}}{M_i^2} = \sum_i (G_3^{nki}G_3^{m\ell i} - G_3^{n\ell i}G_3^{mki})M_i^2,$$

$$\vdots$$

c.f., Sakai and Uekusa, Prog.Theo.Phys. 118 (2007) 315 [hep-th/0604121]

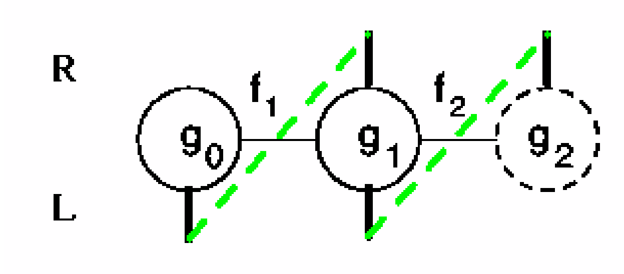
($\tilde{G}_4 = 0$ in the continuum limit)

Low Energy Effective Theory in 4D

How can we ensure the consistency with the existing precision electroweak measurements?

A three-site Higgsless model

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi and M.T., PRD72 075012 (2006);
See also Bando, Kugo, Yamawaki's HLS model Phys.Rep.164,217(1988).



$SU(2) \times SU(2) \times U(1)$ gauge theory

- The gauge sector is precisely that of the BESS model. Casalbuoni et al., PLB 155 95 (1985))
- Fermion mass terms:

$$\mathcal{L}_f = -\lambda f_1 \bar{\psi}_{L0} U_1 \psi_{R1} - M \bar{\psi}_{R1} \psi_{L1} - f_2 \bar{\psi}_{L1} U_2 \begin{pmatrix} \lambda'_u & \\ & \lambda'_d \end{pmatrix} \begin{pmatrix} u_{R2} \\ d_{R2} \end{pmatrix} + \text{h.c.}.$$

- For simplicity, we examine the case $f_1 = f_2 = \sqrt{2}v$ and work in the limit

$$\frac{g_0}{g_1} \ll 1, \quad \frac{g_2}{g_1} \ll 1, \qquad \text{and thus,} \quad g_W \simeq g_0, \quad g_Y \simeq g_2.$$

Fermion mass matrix: (seesaw like)

$$\begin{pmatrix} m & 0 \\ M & m'_f \end{pmatrix} \equiv \sqrt{2}\tilde{\lambda} v \begin{pmatrix} \varepsilon_L & 0 \\ 1 & \varepsilon_{fR} \end{pmatrix}, \qquad \varepsilon_L \equiv \frac{\lambda}{\tilde{\lambda}}, \quad \varepsilon_{fR} \equiv \frac{\lambda'_f}{\tilde{\lambda}}$$

Light fermion mass:

$$m_f \simeq \frac{m m'_f}{\sqrt{M^2 + m'^2_f}} = \frac{\sqrt{2}\tilde{\lambda} v \varepsilon_L \varepsilon_{fR}}{\sqrt{1 + \varepsilon^2_{fR}}}$$

and its eigenstate (or delocalization)

$$\psi_L^{f,\mathrm{light}} \simeq -\left(1 - \frac{\varepsilon_L^2}{2}\right) \psi^f_{L0} + \varepsilon_L \psi^f_{L1}$$

where we assumed $\varepsilon_{fR} \ll 1$.

Heavy (KK) fermion mass:

$$M_{f,KK} \simeq \sqrt{M^2 + m'^2_f} = \sqrt{2}\tilde{\lambda} v \sqrt{1 + \varepsilon^2_{fR}}$$

For $M \gg v$, we can integrate out the heavy KK-fermion. The fermion delocalization effect can then be replaced by an operator

$$\mathcal{L}'_f = -x_1 \bar{\psi}_L (i \not{D} U_1 \cdot U_1^\dagger) \psi_L, \qquad x_1 \equiv \varepsilon_L^2, \quad \varepsilon_L = \frac{\sqrt{2}\lambda v}{M}$$

ψ_L is a left-hand fermion at site-0,

$$D_\mu \psi_L = \partial_\mu \psi_L - i g_0 W_{0\mu} \psi_L.$$

S-parameter

$$S = \frac{4\pi}{g_1^2} \left(1 - \frac{2 g_1^2}{g_0^2} x_1 \right)$$

vanishes in the ideal delocalization limit:

$$x_1 = \frac{g_0^2}{2 g_1^2}, \qquad g_{W'ff} = 0.$$

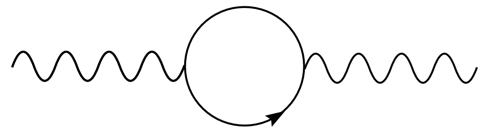
c.f. Anichini, Casalbuoni, and De Curtis, PLB348 521 (1995).

Higgsless confronts electroweak precision tests at one-loop

Matsuzaki, Chivukula, Simmons, and M.T., PRD75, 073002 (2007)
Chivukula, Simmons, Matsuzaki, and M.T., PRD75, 075012 (2007)
Abe, Matsuzaki, and M.T., PRD78, 055020 (2008)
See also, Abe, Chivukula, Christensen, Hsieh, Matsuzaki, Simmons, and M.T., PRD79, 075016 (2009)

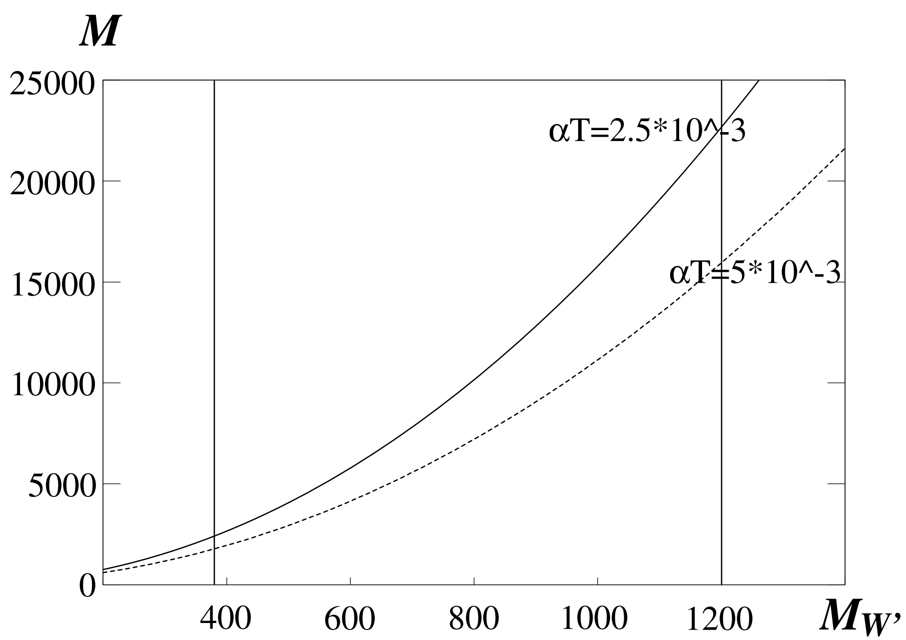
Fermionic one-loop corrections to T parameter

Chivukula, Coleppa, Di Chiara, Simmons, He, Kurachi and M.T., PRD72 075012 (2006)



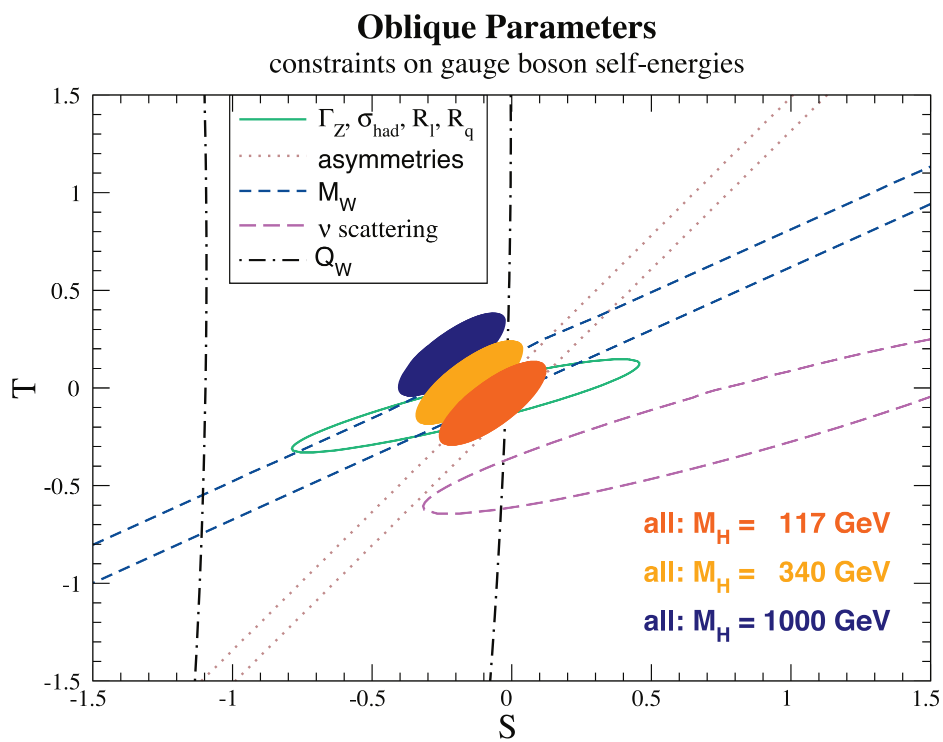
$$\alpha T \approx \frac{1}{16\pi^2}\frac{{m'_t}^4}{M^2 v^2} = \frac{1}{16\pi^2}\frac{\varepsilon_{tR}^4 M^2}{v^2} \ .$$

Assuming ideal delocalization of fermions, we find



Allowed region in S-T depends on the *"reference" Higgs mass* $M_{H,\mathrm{ref}}$.

$$S \equiv S_{\mathrm{BSM}} - S_{\mathrm{SM}}(M_{H,\mathrm{ref}}), \qquad T \equiv T_{\mathrm{BSM}} - T_{\mathrm{SM}}(M_{H,\mathrm{ref}})$$



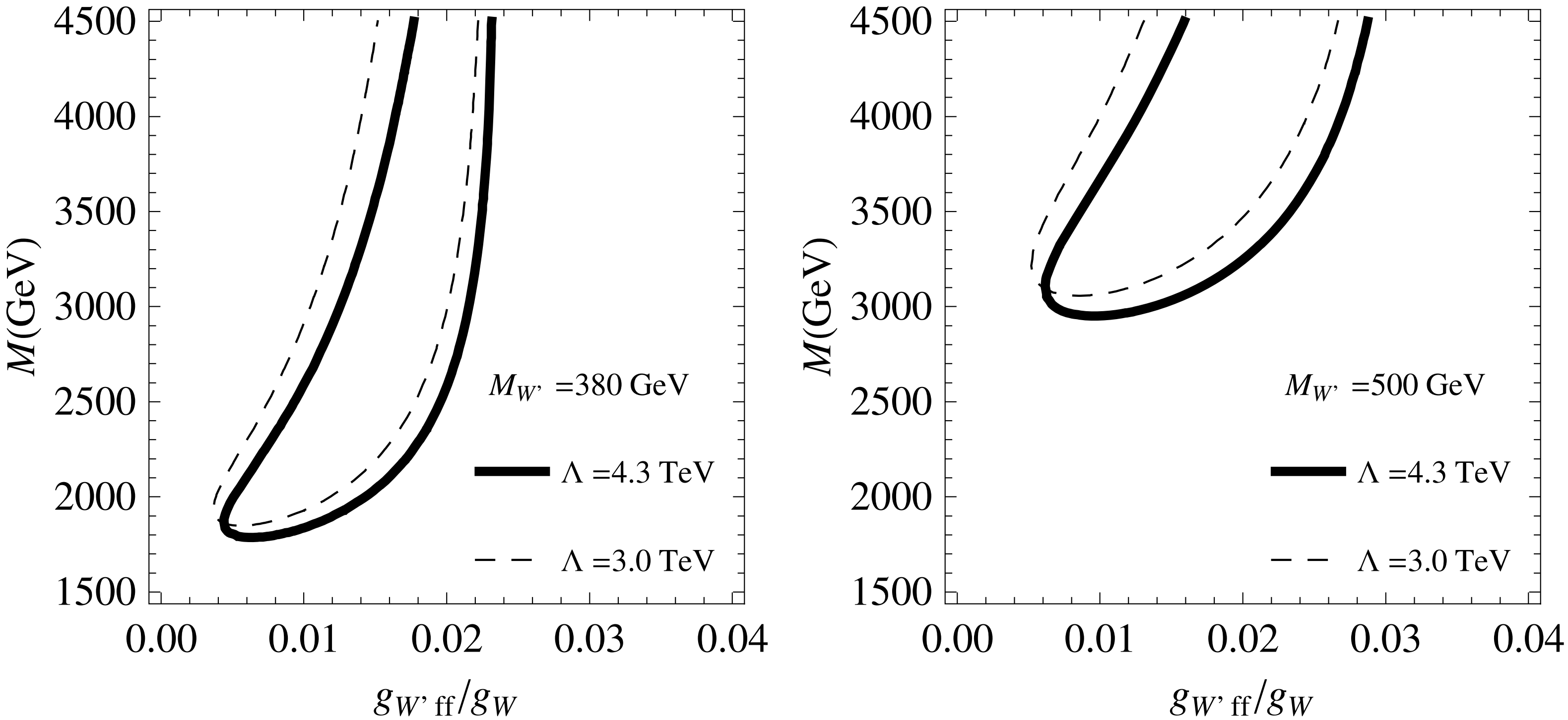
$\alpha T < 2.5 \times 10^{-3}$ (5×10^{-3}) for $M_{H,\mathrm{ref}} = 340$GeV (1000GeV).

Which $M_{H,\mathrm{ref}}$ should we use? *We need to evaluate bosonic one-loop diagrams in order to get more precise bounds.*

Bosonic one-loop contributions to S and T can be evaluated by using the electroweak chiral perturbation technique:

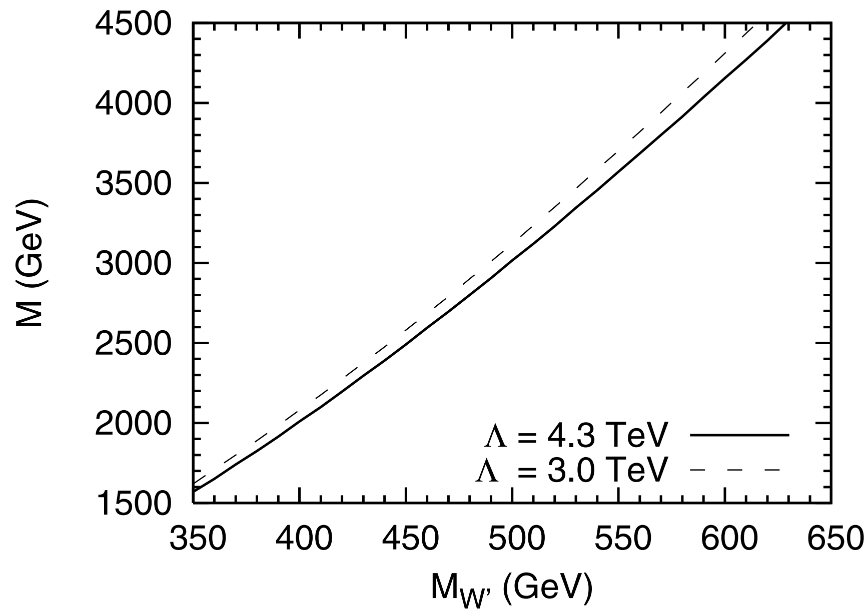
$$\begin{aligned} S &= -4\frac{s^2}{\alpha}\frac{M_W}{M_{W'}}\frac{g_{W'ff}}{g_W} \\ &\quad -\frac{1}{24\pi}\ln\frac{M_{W'}^2}{M^2}+\frac{1}{12\pi}\ln\frac{\Lambda^2}{M_{H,\mathrm{ref}}^2}, \\ T &= -\frac{3}{32\pi c^2}\ln\frac{\Lambda^2}{M_{W'}^2}-\frac{3}{16\pi c^2}\ln\frac{M_{W'}^2}{M_{H,\mathrm{ref}}^2} \\ &\quad +\frac{N_c}{192\pi s^2}\frac{M_{W'}^4}{M_W^6}\frac{m_t^4}{M^2}\left[1+\frac{M_{W'}}{M_W}\frac{g_{W'ff}}{g_W}\right]^{-2}. \end{aligned}$$

One loop constraint from precision electroweak measurements (95%CL):



T. Abe, S. Matsuzaki, and M.T., PRD78, 055020 (2008)

The cutoff dependence is small.
Tiny (but non-zero) $W'ff$ coupling.



- $M_{W'} \gtrsim 380$MeV is required by the ZWW measurement at LEP2.
- The cutoff Λ should satisfy

$$\Lambda \lesssim 4\pi f_1 = 4\pi f_2 = 4.3\text{TeV},$$

which implies

$$M_{W'} \lesssim 600\text{GeV}$$

Higgsless confronts flavor precision tests at one-loop

Abe, Chivukula, Simmons, and M.T., work in progress

Three site fermion sector:

$$\mathcal{L}_f = -f_1 \bar{q}_{L0} U_1 \lambda q_{R1} - \bar{q}_{R1} \mathbf{M} q_{L1} - f_2 \bar{q}_{L1} U_2 \begin{pmatrix} \lambda'_u & \\ & \lambda'_d \end{pmatrix} \begin{pmatrix} u_{R2} \\ d_{R2} \end{pmatrix} + \text{h.c.}.$$

If $\mathbf{M}$ and λ are both flavor universal ($\propto 1$), quark flavor mixings arise solely from λ'_u and λ'_d. $Z\bar{q}q$ and $Z'\bar{q}q$ couplings become flavor diagonal in this case (minimal flavor violation).

Flavor precision tests provides various phenomenological bounds on

$$\delta \equiv \frac{\Delta \mathbf{M}}{M},$$

with

$$\mathbf{M} = \begin{pmatrix} M & & \\ & M & \\ & & M \end{pmatrix} + \Delta \mathbf{M}$$

Constraints from K-$\bar{K}$, B-$\bar{B}$, B_s-$\bar{B}_s$ mixings

$$-9.0 \cdot 10^{-6} < \mathsf{Re}(\delta_{sd})^2 \left(\frac{400 \text{ GeV}}{M_{W'}}\right)^4 < 9.0 \cdot 10^{-6},$$

$$-4.1 \cdot 10^{-8} < \mathsf{Im}(\delta_{sd})^2 \left(\frac{400 \text{ GeV}}{M_{W'}}\right)^4 < 2.6 \cdot 10^{-8},$$

$$\begin{aligned} |\delta_{bd}|^2 \left(\frac{400 \text{ GeV}}{M_{W'}}\right)^4 &< 2.2 \cdot 10^{-4}, \\ |\delta_{bs}|^2 \left(\frac{400 \text{ GeV}}{M_{W'}}\right)^4 &< 1.0 \cdot 10^{-2}. \end{aligned}$$

These constraints should be compared with the one-loop induced size of δ

$$\begin{aligned}\delta_{sd} &= -\frac{1}{(4\pi)^2}\ln\frac{\Lambda}{M}\cdot\frac{M^2}{m_1^2}\frac{m_t^2}{f_2^2}V_{ts}^*V_{td} \\ &\simeq 10^{-10}.\end{aligned}$$

It is easy to satisfy the flavor physics constraints if $\delta = 0$ is imposed at the cutoff scale Λ.

Summary and Outlook

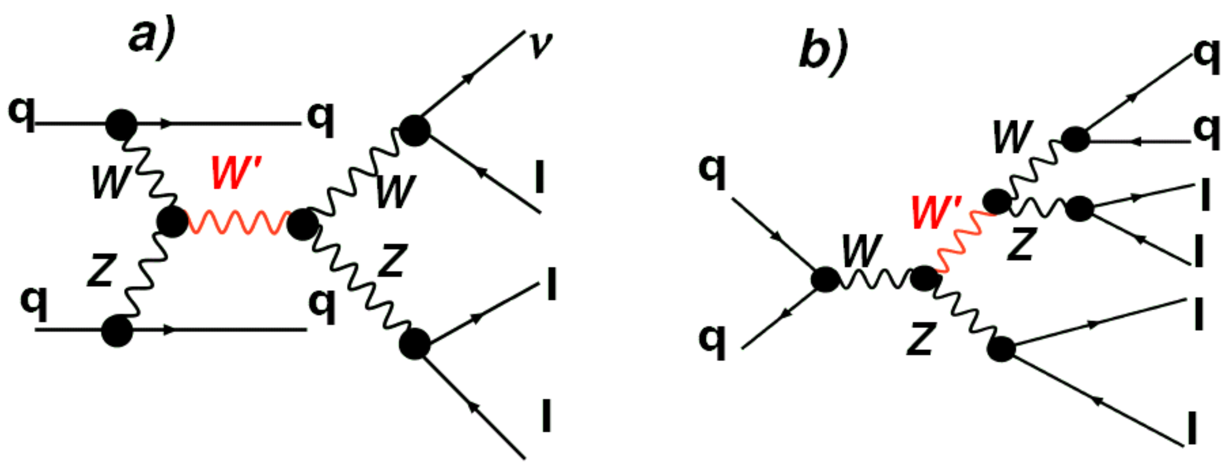
- Higgsless theory is an interesting alternative to the standard model Higgs, achieving tree level unitarity at 1TeV.
- KK W' properties can be determined by unitarity sum rules.
- We analyzed an effective theory (three site Higgsless model) at one-loop level and found the model is consistent with the available precicion electroweak measurements. The allowed ranges of the KK gauge boson coupling $g_{W'ff}$, the KK gauge boson mass $M_{W'}$, and the KK quark/lepton masses M are severely constrained, however.
- The KK gauge boson W' will be discovered at LHC with $\int \mathcal{L} = 20 \sim 30$ fb^{-1}.
- FCNC is protected by GIM mechanism in the case of flavor universal KK-fermion mass. We need more study in this direction.

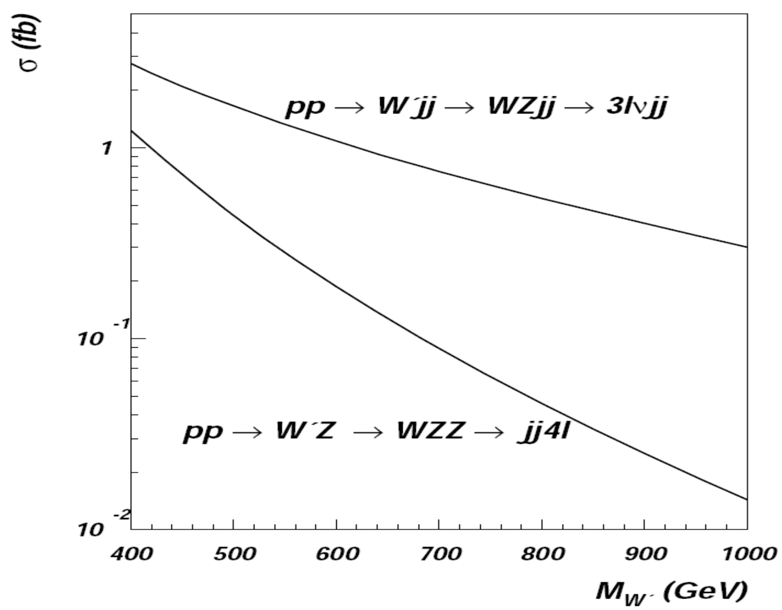
  (work in progress with T. Abe, R.S.Chivukula, and E.H. Simmons)

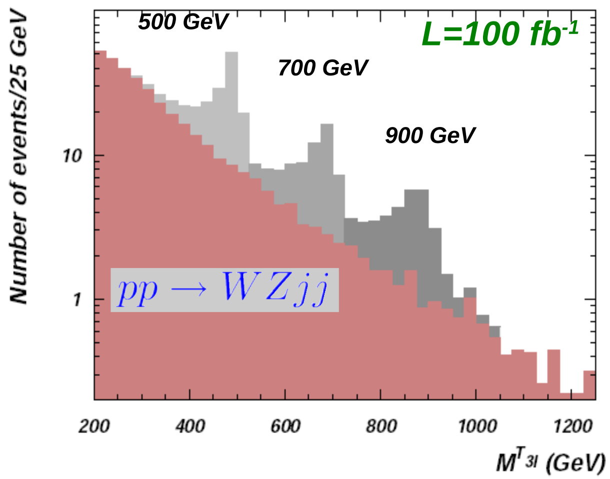
LHC phenomenology of W'

W' production cross sections through $W'WZ$ vertex:

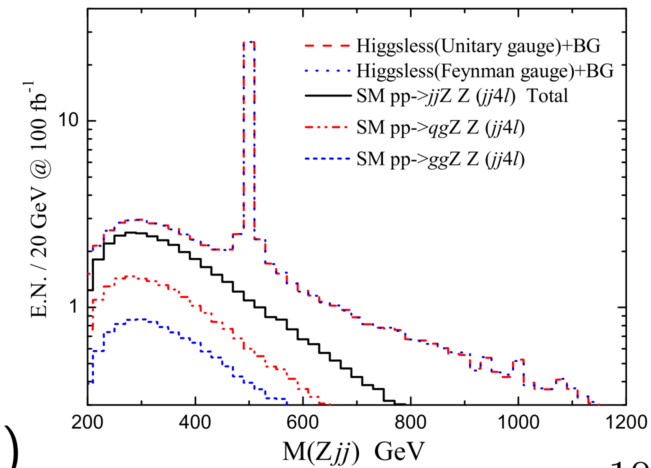
H.-J. He et al., arXiv:0708.2588





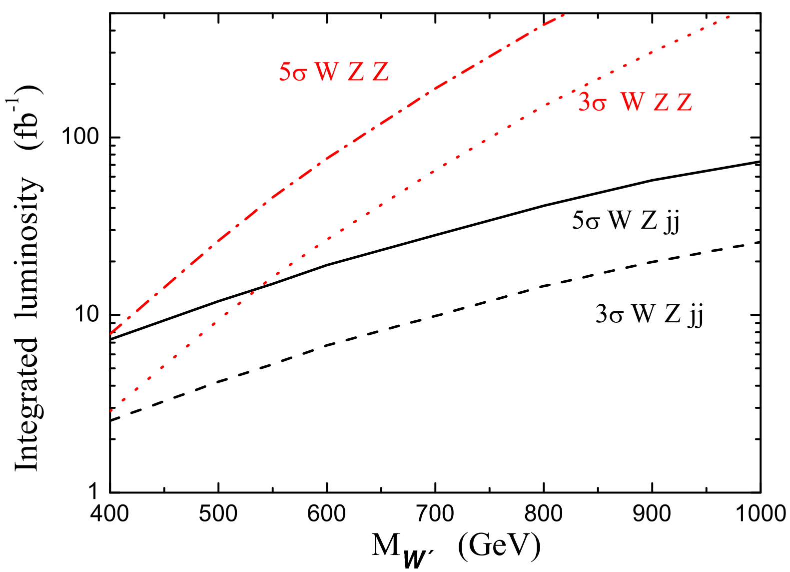


(a)



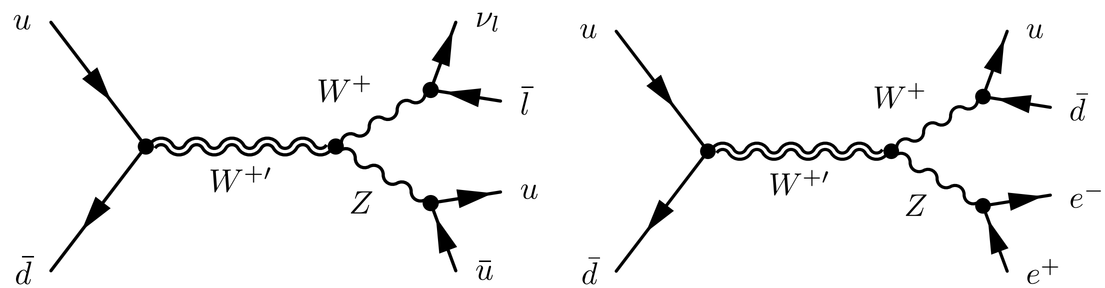
(b)

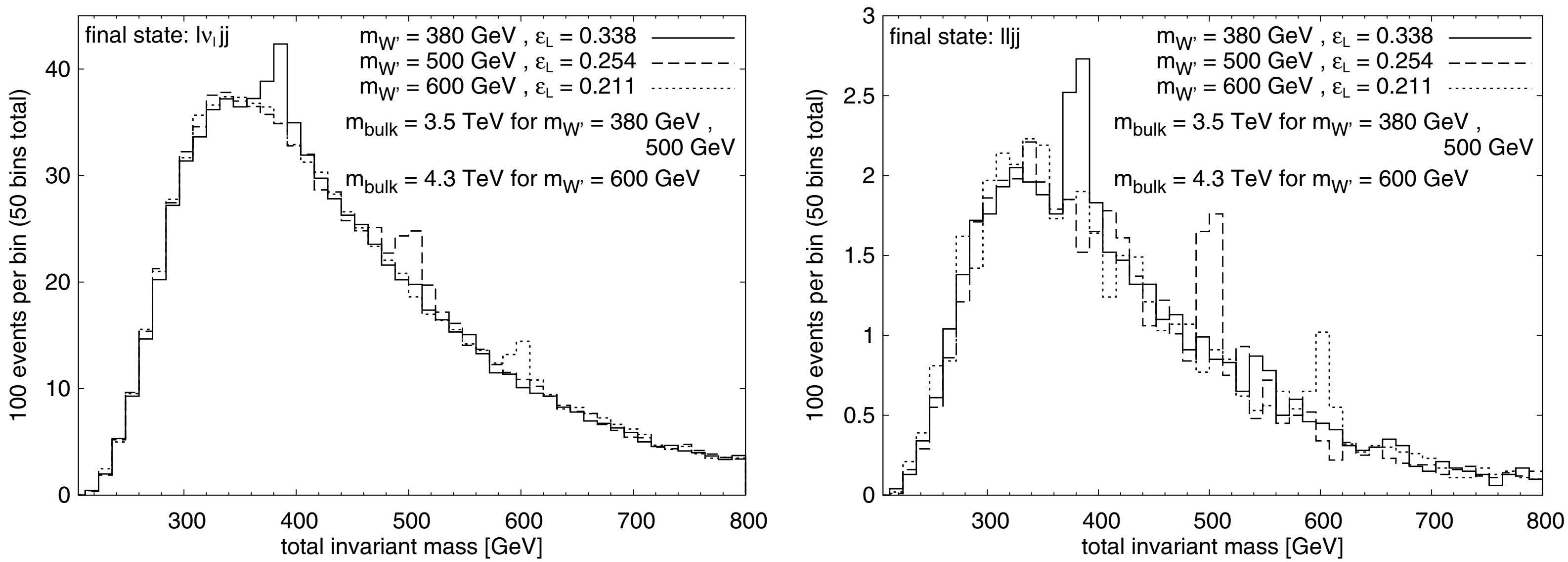
100fb^{-1}



W' production cross sections through $W'ff$ vertex:

T. Ohl and C. Speckner, arXiv:0809.0023





100fb^{-1}