Flavor Mixing in Gauge-Higgs Unification

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Based on the work with
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PLAN

- Introduction
- Model
- $K^0 - \bar{K}^0$ mixing
- Summary

Introduction

In gauge-Higgs unification,
achieving flavor violation is a nontrivial issue

∵ Yukawa couplings are originated from gauge coupling

i.e. universal for all flavors

As a new feature of higher dimensional models with Z_2 orbifold, Z_2-odd bulk masses are allowed

$$M_i \varepsilon(y) \bar{\psi}_i \psi_i \quad (\varepsilon(y): \text{sign function})$$

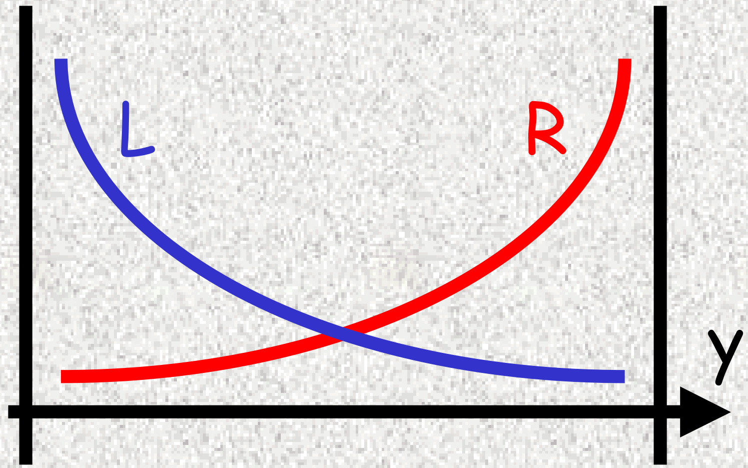
with M_i being different depending on each flavor

↓

New source of flavor violation specific to higher dimensional models

Zero mode Weyl fermions
with different chirality localize
at an opposite side

$$f_L^{(0)}(y) = \sqrt{\frac{M_i}{1-e^{-2\pi M_i R}}}\, e^{-M_i|y|},\quad f_R^{(0)}(y) = \sqrt{\frac{M_i}{e^{2\pi M_i R}-1}}\, e^{M_i|y|}$$



4D effective Yukawa coupling

$$Y = g_4 \int_{-\pi R}^{\pi R} dy f_L^{(0)}(y)\, f_R^{(0)}(y) \approx 2\pi M R g_4 e^{-\pi M R} \le g_4$$

No need of unnatural fine-tuning for 5D parameters
due to exponential suppression
To get a viable top mass is not trivial, but possible

At first glance,
the bulk masses can be off-diagonal in flavor space,
which seems to generate flavor mixing

Unfortunately, it is not the case:
For each representation R of the gauge group,
a general form of bulk mass terms

$$M(R)_{ij}\,\varepsilon(y)\bar{\psi}(R)_i\,\psi(R)_j$$

can be diagonalized
by a suitable unitary transformation,
leaving the kinetic term invariant

We are led to introduce

brane localized mass terms,

which are necessary to make exotics heavy

& are the sources of flavor mixing

as will be seen below

Model

5D SU(3) model compactified on S^1/Z_2

N-generations of bulk fermion are introduced

$$\psi^i(3) = Q_3^i \oplus d^i$$

$$\psi^i(\overline{6}) = \Sigma^i \oplus Q_6^i \oplus u^i$$

(i = 1, …, N)

Need to eliminate the redundant quark doublets (Q) and exotics (Σ)

→ Brane localized mass terms

$$\mathcal{L} = -\frac{1}{4}F^{MN}F_{MN} + \bar{\psi}_3^i\left(i\not{D} - M^i\varepsilon(y)\right)\psi_3^i + \bar{\psi}_{\bar{6}}^i\left(i\not{D} - M^i\varepsilon(y)\right)\psi_{\bar{6}}^i$$

"generation dependent" bulk masses

$$+\delta(y)\sqrt{2\pi R}\bar{Q}_R^i(x)\left[\eta_{ij}Q_{3L}^j(x,y) + \lambda_{ij}Q_{\bar{6}L}^j(x,y)\right] + \cdots$$

Brane localized fields

Brane mass matrices (off-diagonal elements are generically allowed)

→ "Flavor mixing"

$$\mathcal{L}_{BM}^{Q} \sim \delta(y)\bar{Q}_R[\eta \;\; \lambda]\begin{bmatrix} Q_3 \\ Q_6 \end{bmatrix}_L = \delta(y)\bar{Q}'_R\left[m_{diag} \;\; 0\right]\begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L$$

$$\begin{bmatrix} Q_3 \\ Q_6 \end{bmatrix}_L = \begin{bmatrix} U_1 & U_3 \\ U_2 & U_4 \end{bmatrix}\begin{bmatrix} Q_H \\ Q_{SM} \end{bmatrix}_L, \quad U^{\bar{Q}}Q_R = Q'_R$$

"2Nx2N unitary matrix"

Yukawa coupling

$$\mathcal{L}_{Yukawa} = g_5 A_y^6 \bar{d}^i Q_3^i + g_5 A_y^6 \bar{u}^i Q_6^i$$
$$\supset g_5 A_y^6 \bar{d}^i U_3^{ij} Q_{SM}^j + g_5 A_y^6 \bar{u}^i U_4^{ij} Q_{SM}^j$$
$$\to g_5 \left\langle A_y^6 \right\rangle \left(\bar{d}_R^{i(0)} Y_d^{ii} U_3^{ij} Q_{SM}^{j(0)} + \bar{u}_R^{i(0)} Y_u^{ii} U_4^{ij} Q_{SM}^{j(0)} \right)$$

Yukawa coupling with flavor mixing

$$Y^{ii} = \int_{-\pi R}^{\pi R} dy f_L^{i(0)} f_R^{i(0)} \approx 2\pi R M^i e^{-\pi R M^i}$$

Diagonalization

$$\begin{cases} \hat{Y}_d = V_{dR}^{\dagger} Y_d U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^{\dagger} Y_u U_4 V_{uL} \end{cases} \quad V_{CKM} = V_{uL}^{\dagger} V_{dL} \quad \left(U_3^{\dagger} U_3 + U_4^{\dagger} U_4 = 1_{N\times N} \right)$$

$M_{3,6} \propto 1$ ($Y_{u,d} \propto 1$) case (flavor symmetry restored)

$$\begin{cases} \hat{Y}_d = V_{dR}^\dagger U_3 V_{dL} \rightarrow \hat{Y}_d^2 = V_{dL}^\dagger U_3^\dagger U_3 V_{dL} \\ \hat{Y}_u = V_{uR}^\dagger U_4 V_{uL} \rightarrow \hat{Y}_u^2 = V_{uL}^\dagger U_4^\dagger U_4 V_{uL} \end{cases} \xrightarrow{U_3^\dagger U_3 + U_4^\dagger U_4 = 1} V_{uL} \propto V_{dL}$$

$$\Rightarrow V_{CKM} = V_{uL}^\dagger V_{dL} \propto V_{dL}^\dagger V_{dL} = 1 \text{(No mixing)}$$

Lesson

To get flavor mixing,
we need non-degenerate bulk masses
as well as the off-diagonal brane masses
(specific to gauge-Higgs unification)

Now, we focus on 2 generation case

Parameterize rotation angles as (CP invariance is assumed)

$$U_4 = \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad U_3 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix}$$

with satisfying a unitarity condition $U_3^\dagger U_3 + U_4^\dagger U_4 = 1$

Yukawa couplings

$$\hat{Y}_d = V_{dR}^\dagger I_{RL}^{(00)} U_3 V_{dL} = V_{dR}^\dagger \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix} V_{dL}$$

$$\hat{Y}_u = V_{uR}^\dagger I_{RL}^{(00)} U_4 V_{uL} = V_{uR}^\dagger \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{bmatrix} \cos\theta' & -\sin\theta' \\ \sin\theta' & \cos\theta' \end{bmatrix} \begin{bmatrix} a^2 & 0 \\ 0 & b^2 \end{bmatrix} V_{uL}$$

$$I_{RL}^{(00)} = \int_{-\pi R}^{\pi R} dy f_L^{i(0)} f_R^{i(0)} \approx 2\pi R M^i e^{-\pi R M^i}$$

Parameter fitting (6 parameters - 5 observables = 1 parameter)

$$\begin{cases} \hat{m}_u \hat{m}_c = \det \hat{Y}_u ,\ \hat{m}_d \hat{m}_s = \det \hat{Y}_d ,\ \hat{m}_i \equiv m_i / m_W \\ \hat{m}_u^2 + \hat{m}_c^2 = Tr \hat{Y}_u \hat{Y}_u^\dagger ,\ \hat{m}_d^2 + \hat{m}_s^2 = Tr \hat{Y}_d \hat{Y}_d^\dagger ,\ \theta_c = \theta_{dL} - \theta_{uL} \end{cases}$$

$$\hat{m}_u^2 \hat{m}_c^2 = a^2 b^2 c^2 d^2 ,\ \hat{m}_d^2 \hat{m}_s^2 = \left(1-a^2\right)\left(1-b^2\right) c^2 d^2$$

$$\hat{m}_u^2 + \hat{m}_c^2 = a^2 c^2 + b^2 d^2 - \left(a^2 - b^2\right)\left(c^2 - d^2\right)\sin^2\theta'$$

$$\hat{m}_d^2 + \hat{m}_s^2 = \left(1-a^2\right)c^2 + \left(1-b^2\right)d^2 - \left(a^2 - b^2\right)\left(c^2 - d^2\right)\sin^2\theta$$

$$\tan 2\theta_c = \frac{\tan 2\theta_{dL} - \tan 2\theta_{uL}}{1 + \tan 2\theta_{dL} \tan 2\theta_{uL}}$$

$$\begin{cases} \tan 2\theta_{dL} = \dfrac{2\sqrt{\left(1-a^2\right)\left(1-b^2\right)}\left(d^2 - c^2\right)\sin\theta\cos\theta}{\left(1-a^2\right)\left(c^2\cos^2\theta + d^2\sin^2\theta\right) - \left(1-b^2\right)\left(c^2\sin^2\theta + d^2\cos^2\theta\right)} \\ \tan 2\theta_{uL} = \dfrac{2ab\left(d^2 - c^2\right)\sin\theta'\cos\theta'}{a^2\left(c^2\cos^2\theta' + d^2\sin^2\theta'\right) - b^2\left(c^2\sin^2\theta' + d^2\cos^2\theta'\right)} \end{cases}$$

Mixing again

$$\tan 2\theta_c = \frac{\tan 2\theta_{dL} - \tan 2\theta_{uL}}{1 + \tan 2\theta_{dL} \tan 2\theta_{uL}} \propto \left(d^2 - c^2\right)$$

$$\begin{cases} \tan 2\theta_{dL} = \dfrac{2\sqrt{\left(1-a^2\right)\left(1-b^2\right)}\left(d^2 - c^2\right)\sin\theta\cos\theta}{\left(1-a^2\right)\left(c^2\cos^2\theta + d^2\sin^2\theta\right) - \left(1-b^2\right)\left(c^2\sin^2\theta + d^2\cos^2\theta\right)} \\ \tan 2\theta_{uL} = \dfrac{2ab\left(d^2 - c^2\right)\sin\theta'\cos\theta'}{a^2\left(c^2\cos^2\theta' + d^2\sin^2\theta'\right) - b^2\left(c^2\sin^2\theta' + d^2\cos^2\theta'\right)} \end{cases}$$

Check

Universal bulk mass limit (c=d)

→ $\tan 2\theta_c = 0$ i.e. $\theta_c = 0$

Natural flavor conservation

FCNC has played crucial roles
in the discussion of the viability of New Physics

We ask if "natural flavor conservation" is satisfied,
i.e. if FCNC processes at tree level are forbidden

Glashow-Weinberg condition

"Fermions with the same electric charges, chirality
should have the same quantum numbers
(such as I_3)"

Glashow & Weinberg (1977)

The condition is satisfied in the down sector relevant to K-Kbar mixing @tree level

Parity assignment

$$\mathbf{3} = \begin{cases} \mathbf{2}_{L1/6}\ (Q)\ (+) + \mathbf{1}_{L-1/3}\ (-) \\ \mathbf{2}_{R1/6}\ (-) + \mathbf{1}_{R-1/3}\ (d_R)\ (+) \end{cases}$$

$$\mathbf{6}^* = \begin{cases} \mathbf{3}_{L-1/3}\ (-) + \mathbf{2}_{L1/6}\ (Q)\ (+) + \mathbf{1}_{L2/3}\ (-) \\ \mathbf{3}_{R-1/3}\ (+) + \mathbf{2}_{R1/6}\ (-) + \mathbf{1}_{R2/3}\ (u_R)\ (+) \end{cases}$$

"exotic"

$$3_{\uparrow R}\,(2/3) \oplus 3_{0R}\,(-1/3) \oplus 3_{\downarrow R}\,(-4/3)$$

Same quantum number as down quark

"Not" the end of the story

$K^0 - \bar{K}^0$ mixing

Non-degenerate bulk masses

⇒ FCNC at tree level even in QCD sector

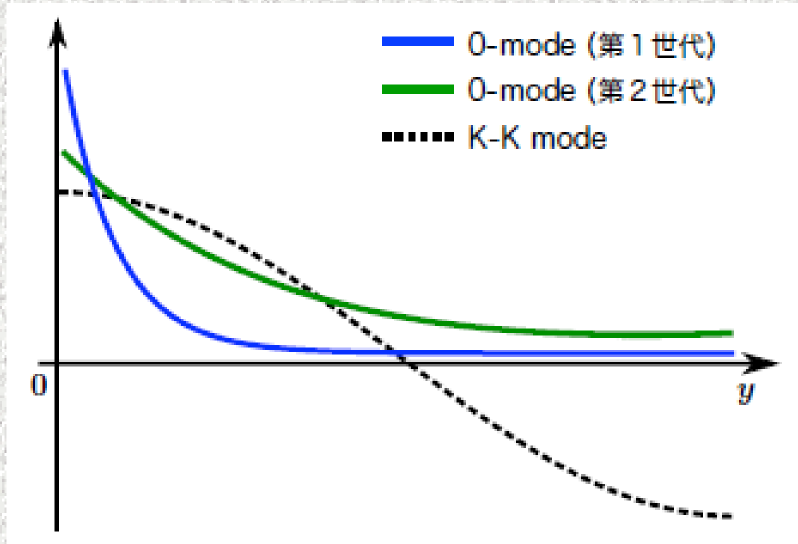
$$\mathcal{L}_{strong} \supset \frac{g_s}{\sqrt{2\pi R}} G_\mu^{a(0)} \left(\bar{d}_R^{i(0)} \gamma^\mu \lambda^a d_R^{i(0)} + \bar{d}_L^{i(0)} \gamma^\mu \lambda^a d_L^{i(0)} \right)$$

$$+ g_s G_\mu^{a(n)} \bar{d}_R^{i(0)} \gamma^\mu \lambda^a d_R^{j(0)} \left(V_{dR}^\dagger I_{RR}^{(0n0)} V_{dR} \right)_{ij}$$

0 mode sector: No mixing O.K.

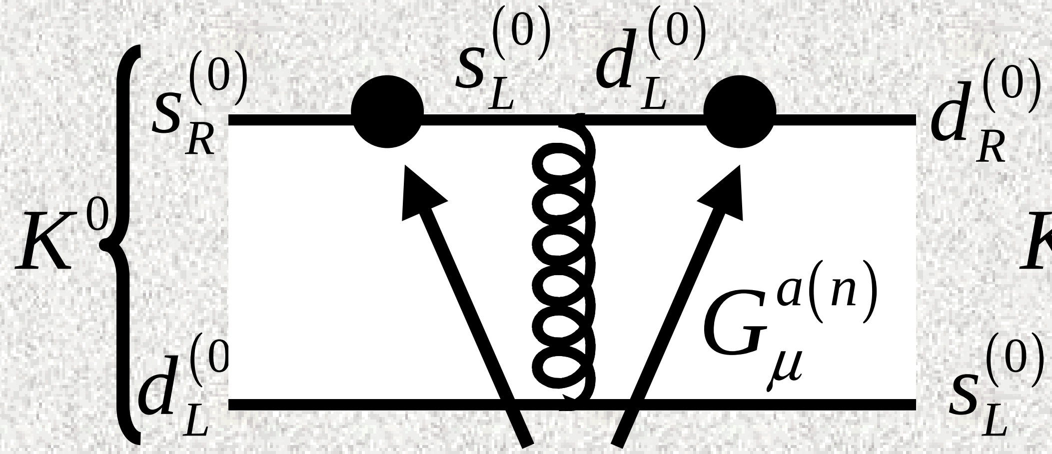
Nonzero KK gluon couplings induce nontrivial flavor mixing

⇒ $K^0 - \bar{K}^0$ mixing@tree level



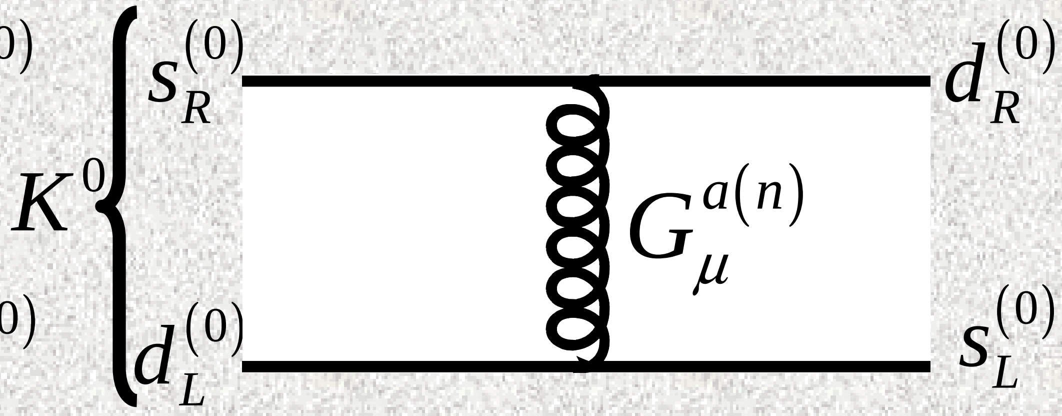
3 types of amplitudes should be considered

Left-Left type



Mass insertion is necessary

Left-Right type



$$K^0 \sim d\gamma^5\bar{s}$$

"Dominant"

"Chiral suppression"

$$\left(\frac{m_d+m_s}{m_K}\right)^2 \approx \left(\frac{106MeV}{497MeV}\right)^2 \approx \left(\frac{1}{5}\right)^2$$

(Right-Right type is also suppressed similarly)

K_L-K_S mass difference

We have calculated K_L-K_S mass difference from the dominant left-right type amplitude

$$\Delta m_K\,(\text{KK modes}) = 2\left\langle \bar{K} \right| \mathcal{L}_{eff}^{\Delta S=2} \left| K \right\rangle$$

$$\sim -2\pi^2 \alpha_S C R^3 \left(B_4 - \frac{B_5}{9} \right) \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$

$$\sim 7.5\times 10^4 \left(R f_\pi \right)^2 C \pi R \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2$$

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \left(f_R^i(y) \right)^2 \cos\left(\frac{n}{R} y \right)$$

$$C = -\frac{1}{2}\left(1-a^2\right) \sin 2\theta_{dL} \cos^2\theta + \frac{1}{2}\left(1-b^2\right) \sin 2\theta_{dL} \sin^2\theta - \frac{1}{2}\sqrt{\left(1-a^2\right)\left(1-b^2\right)} \cos 2\theta_{dL} \sin 2\theta$$

$$-\frac{1}{2} a^2 \sin 2\theta_{dL} \cos^2\theta' + \frac{1}{2} b^2 \sin 2\theta_{dL} \sin^2\theta' - \frac{1}{2} ab \cos 2\theta_{dL} \sin 2\theta'$$

$B_{4,5}$: Bag parameters (B_4=0.81, B_5=0.56)

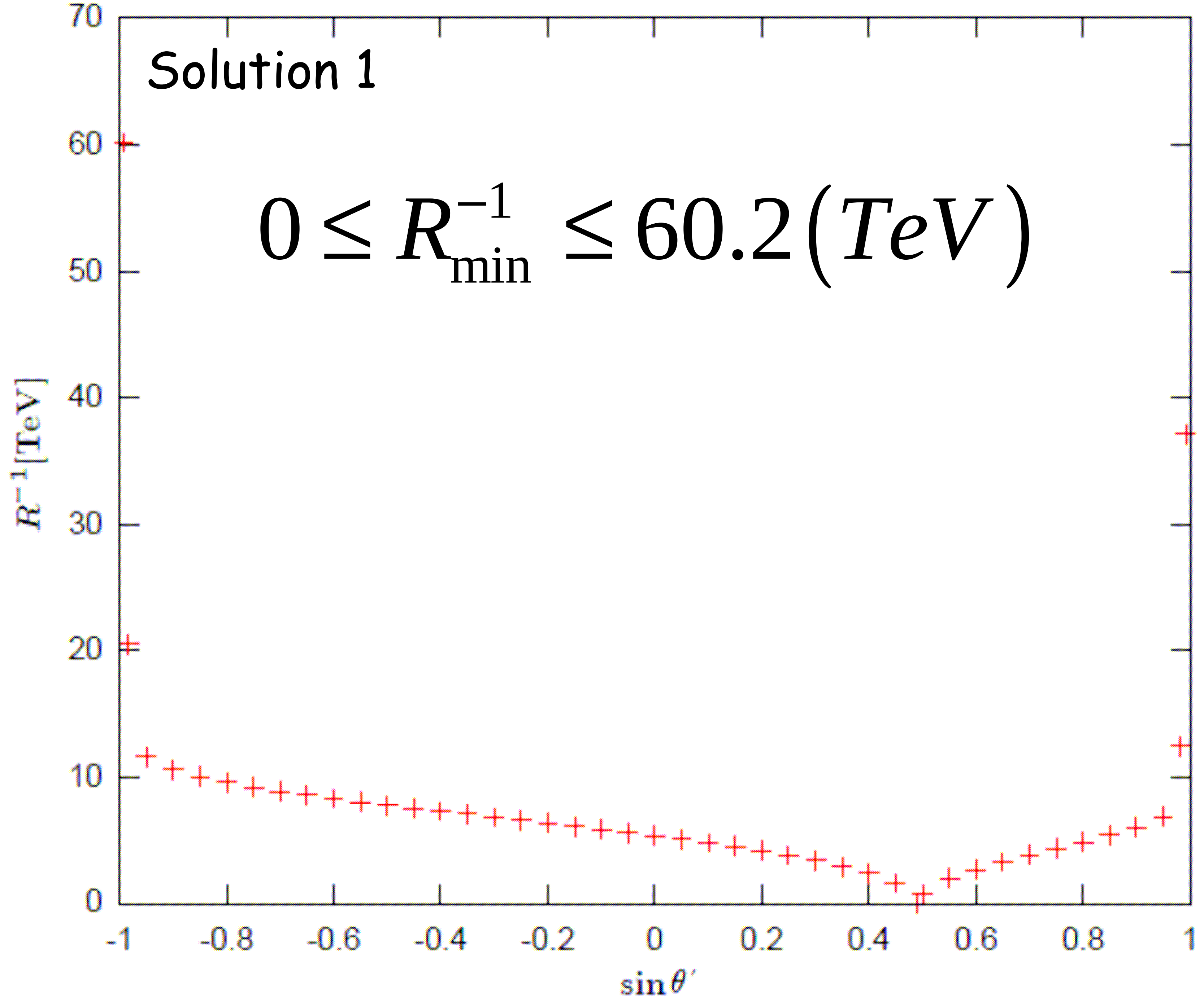
Comparing the data

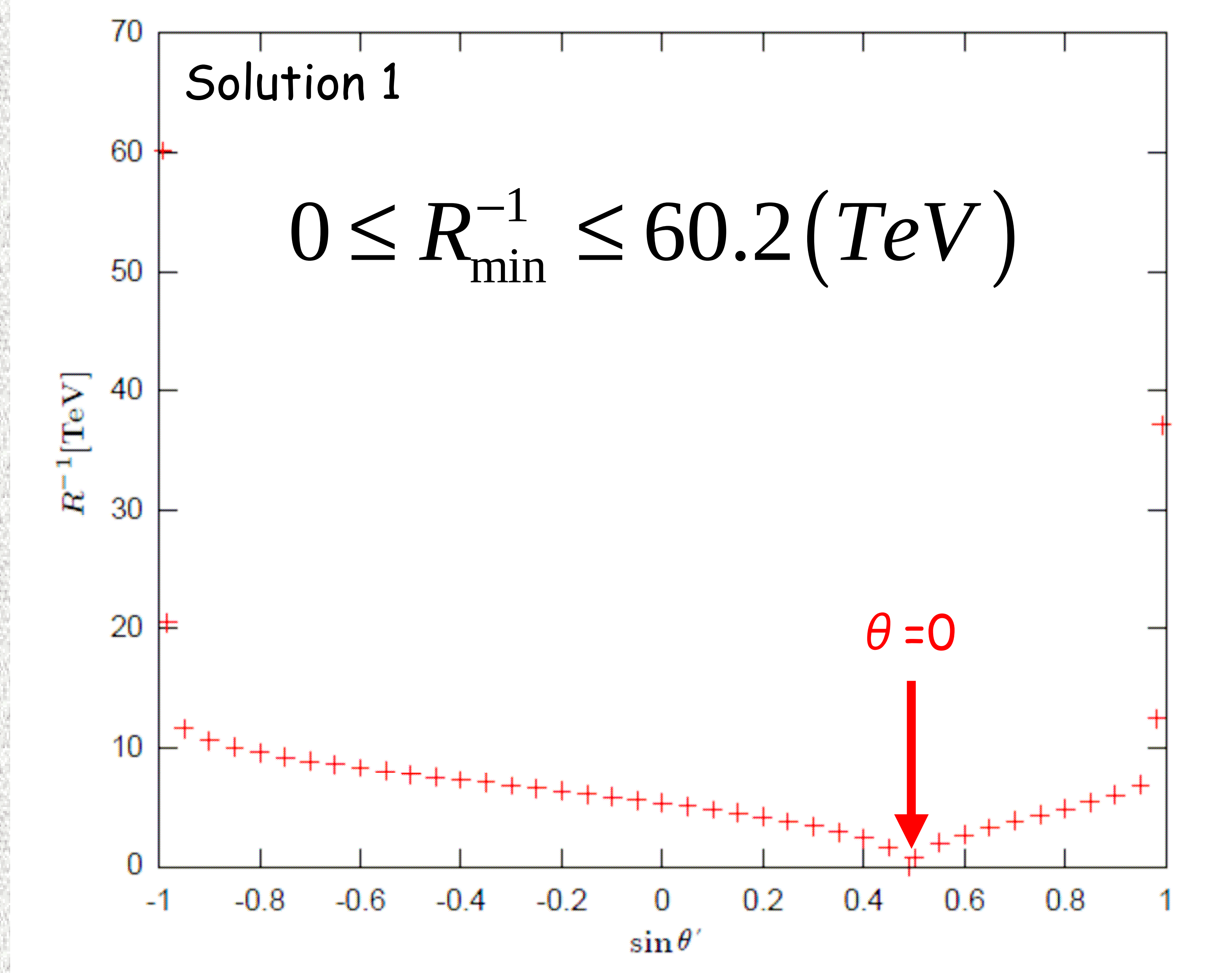
Δm_K (KK modes) < 5.8 x 10^(-13) MeV

gives the lower bound
for the compactification scale as

$$\frac{1}{R} > \sqrt{\sin 2\theta_{dR} C\pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2} \text{ TeV}$$

See numerical plots


Solution 1
$0 \le R_{\min}^{-1} \le 60.2(TeV)$
70
60
50
40
30
20
10
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
R^{-1}[TeV]
$\sin\theta'$


Solution 1
$0 \le R_{\min}^{-1} \le 60.2(TeV)$
θ =0
R^{-1}[TeV]
sin θ′
70
60
50
40
30
20
10
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

Comment on a point 1/R=0

$$U_3 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix} \xrightarrow{\theta=0} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix}$$

$$\hat{Y}_d = V_{dR}^{\dagger} \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} \begin{bmatrix} \sqrt{1-a^2} & 0 \\ 0 & \sqrt{1-b^2} \end{bmatrix} V'_{dL},$$

Vanishing mixing in down sector

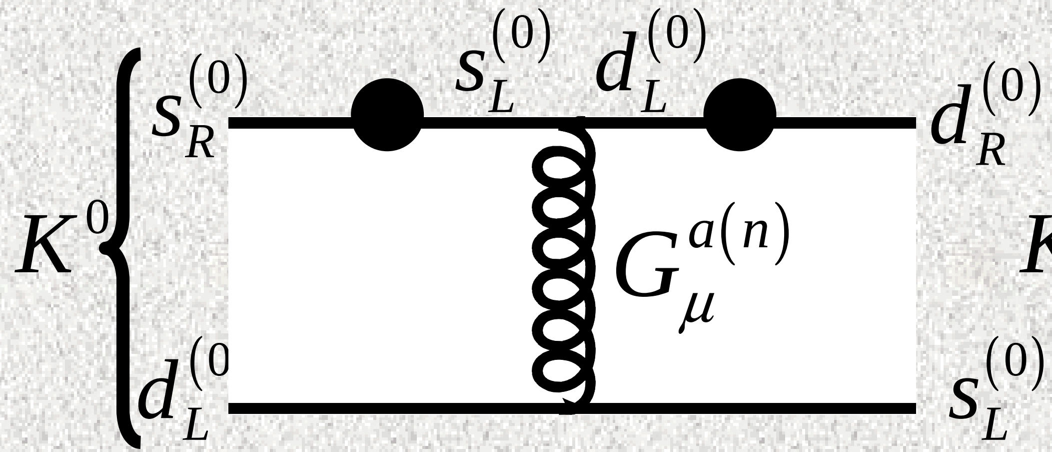
$$\therefore \hat{Y}_d \hat{Y}_d^{\dagger} = V_{dR}^{\dagger} \begin{bmatrix} \left(1-a^2\right) c^2 & 0 \\ 0 & \left(1-b^2\right) d^2 \end{bmatrix} V_{dR} \Rightarrow V_{dR} = 1, \; i.e. \; \theta_{dR} = 0$$

$$\Delta m_K \left(\text{KK modes}\right) \sim \alpha_S R^2 f_K^2 m_K \sin 2\theta_{dR} \sum_n \frac{(-1)^n}{n^2} \left[I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right]^2 = 0$$

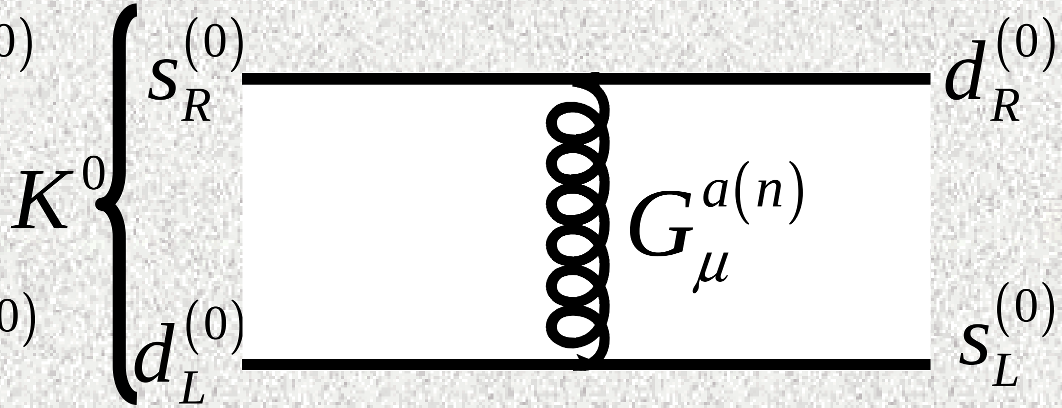
No constraint of 1/R from the left-right type…

But, we have to notice that the L-L type or R-R type dominates over the L-R one at some value of sin θ'

Left-Left type



Left-Right type



$$K^0 \sim \bar{s}\gamma^5 d$$

Chiral suppression

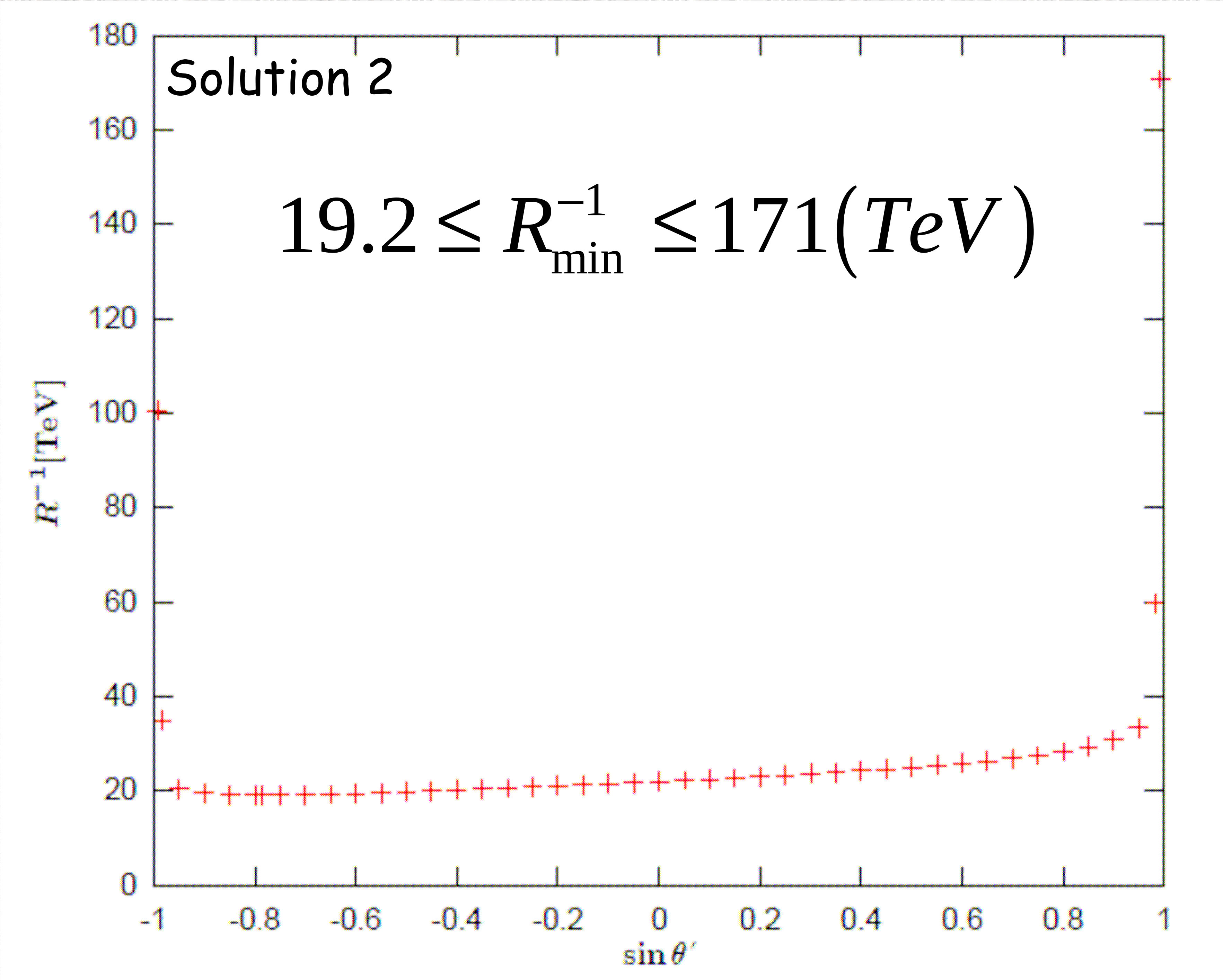
$$\left(\frac{m_d + m_s}{m_K}\right)^2 \approx \left(\frac{106MeV}{497MeV}\right)^2 \approx \left(\frac{1}{5}\right)^2$$

(Right-Right type is also suppressed similarly)

Most stringent lower bound of 1/R from LL

~ 60TeV x 1/5

~ O(10TeV)



Solution 2
$19.2 \le R_{\min}^{-1} \le 171(TeV)$
R^{-1}[TeV]
$\sin\theta'$
180
160
140
120
100
80
60
40
20
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

"GIM-like" mechanism in GHU

In the above results,
the lower bound for the compactification scale is smaller than that from naïve order estimate
(except the extreme case of $|\sin\theta'|\sim 1$)

$$\frac{(\sin\theta_c \cos\theta_c)^2}{M_{KK}^2} \leq \frac{1}{(10^5 TeV)^2} \Rightarrow M_{KK} \geq 300 TeV$$

This apparent discrepancy can be understood since the "GIM-like" mechanism works in GHU

i.e. FCNC processes are automatically suppressed for light generation of quarks

Light quarks masses are obtained from the large bulk masses through the factor exp[- π RM]

In the large bulk mass limit, the KK mode sum can be approximated as follows

$$S_{KK} \equiv \pi R \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2$$

$$\simeq -\frac{\pi^2}{2} \left(e^{-2\pi R M^1} + e^{-2\pi R M^2} \right)$$

$$-\frac{\pi}{2R} \frac{\left(M^1\right)^2 - M^1 M^2 + \left(M^2\right)^2}{M^1 M^2 \left(M^1 - M^2\right)} \left(e^{-2\pi R M^1} - e^{-2\pi R M^2} \right) \left(\pi R M^i \gg 1 \right)$$

exponential suppression!!

$$e^{-2\pi R M^i} \Leftrightarrow \frac{m_{q^i}^2}{m_W^2}$$

similar to GIM suppression

$$\frac{m_c^2 - m_u^2}{m_W^2}$$

More intuitive understanding of "GIM-like" suppression

FCNC is controlled by the factor

$$\left(I_{RR}^{1(0n0)} - I_{RR}^{2(0n0)} \right)^2$$

$$I_{RR}^{i(0n0)} = \frac{1}{\sqrt{\pi R}} \int_{-\pi R}^{\pi R} dy \frac{M^i}{e^{2\pi R M^i} - 1} e^{2M^i y} \cos\left(\frac{n}{R} y \right)$$

In $\pi MR \gg 1$ limit & for small mode index n

Width "1/M" of 0 mode function $\ll$ Period $2\pi R/n$ of KK gluon mode function

Almost flat KK gluon mode function
for fast exponential dumping 0 mode fermions

⇒ Almost flavor universal (similar to 0 mode sector)

In the extreme case of $|\sin\theta'| \sim 1$
the bulk mass of 2nd generation
happens to be relatively small
(see plot, $d \sim 1$)

↓

"GIM-like" mechanism does not work

↓

Severe lower bound for the compactification scale

$\sin\theta'$	a^2	b^2	c^2	d^2	$\sin\theta$
0.9999	0.0000578	0.99999	4.18×10^{-9}	1.000000	0.000341
0.9	0.0526	0.9986	4.28×10^{-9}	0.001074	0.008414
0.8	0.0927	0.9974	4.37×10^{-9}	0.000597	0.008562
0.7	0.1236	0.9964	4.46×10^{-9}	0.000439	0.006756
0.6	0.1473	0.9956	4.55×10^{-9}	0.000362	0.003876
0.5	0.1652	0.9950	4.63×10^{-9}	0.000317	0.000345
0.4909	0.1665	0.9950	4.64×10^{-9}	0.000314	0.000000
0.4	0.1781	0.9945	4.72×10^{-9}	0.000289	-0.003569
0.3	0.1868	0.9942	4.80×10^{-9}	0.000271	-0.007677
0.2	0.1919	0.9940	4.88×10^{-9}	0.000259	-0.011827
0.1	0.1935	0.9940	4.96×10^{-9}	0.000253	-0.015892
0.0	0.1919	0.9940	5.04×10^{-9}	0.000251	-0.019758
-0.1	0.1873	0.9942	5.12×10^{-9}	0.000253	-0.023314
-0.2	0.1797	0.9945	5.20×10^{-9}	0.000259	-0.026451
-0.3	0.1691	0.9949	5.29×10^{-9}	0.000271	-0.029052
-0.4	0.1555	0.9954	5.38×10^{-9}	0.000290	-0.030985
-0.5	0.1387	0.9959	5.47×10^{-9}	0.000319	-0.032090
-0.6	0.1186	0.9966	5.57×10^{-9}	0.000367	-0.032156
-0.7	0.0950	0.9974	5.67×10^{-9}	0.000450	-0.030871
-0.8	0.0676	0.9982	5.77×10^{-9}	0.000620	-0.027690
-0.9	0.0360	0.9991	5.88×10^{-9}	0.001140	-0.021342
-0.9999	0.0000402	0.99999	6.00×10^{-9}	1.000000	-0.000747

In the extreme case of $|\sin\theta'| \sim 1$
the bulk mass of 2^{nd} generation
happens to be relatively small
(see plot, $d \sim 1$)
↓
"GIM-like" mechanism does not work
(No exponential suppression)
↓
Severe lower bound for the compactification scale

Summary

We have studied the mechanism of generating flavor mixing in gauge-Higgs Unification

- Non-degenerate bulk masses as well as brane masses play an important role for flavor mixing
- Especially, non-degenerate bulk masses are new sources of flavor violation beyond the Glashow-Weinberg argument & lead to FCNC at tree level
- In the case of $K^0 - \bar{K}^0$ mixing, nonzero KK gluon exchange at tree level yields the amplitude suppressed by the compactification scale and the data put its lower bound like O(10TeV)
- "GIM-like" mechanism works in GHU as well

Future directions

- Application to $D^0 - \bar{D}^0$ mixing (in progress)
- Extension to 3 generations
  ($B^0 - \bar{B}^0$ mixing, b→sγ etc)
- Application to lepton sector (in progress)
  (μ→3e, μ→e conversion, μ→eγ etc)

Backup Slides

No reason to choose the same bulk masses for different representations, 3 & 6*

Natural choice if we have some GUT where 3 & 6* are embedded into a single representation of the GUT group

$Sp(8) \to Sp(6) \times SU(2) \to SU(3) \times U(1) \times SU(2)$

$$36 \to (1, 3) + (21, 1) + (6, 2)$$
$$\to (1, 3) + (1 + 6 + \mathbf{6^*} + 8, 1) + (\mathbf{3} + 3^*, 2)$$

3 & 6* of SU(3) can be embedded into an adjoint representation 36 of Sp(8)

Lagrangian

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left(F_{MN}F^{MN}\right) + i\bar{\Psi}\Gamma^{M}D_{M}\Psi \qquad \Gamma^{M} = \left(\gamma^{\mu}, i\gamma^{5}\right)$$

$$F_{MN} = \partial_{M}F_{N} - \partial_{N}F_{M} - ig_{D+1}\left[A_{M}, A_{N}\right]\left(M, N = 0,1,2,3,5\right)$$

$$D_{M} = \partial_{M} - ig_{5}A_{M}\left(A_{M} = A_{M}^{a}\,\lambda^{a}/2 : \lambda^{a} : \text{Gell-Mann matrices}\right)$$

$$\Psi = \left(\psi_{1}, \psi_{2}, \psi_{3}\right)^{T}$$

Boundary conditions:

$$A_{\mu} = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix}, A_{D+1} = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}, \Psi = \begin{pmatrix} \psi_{1L}(+,+) + \psi_{1R}(-,-) \\ \psi_{2L}(+,+) + \psi_{2R}(-,-) \\ \psi_{3L}(-,-) + \psi_{3R}(+,+) \end{pmatrix}$$

SU(3) → SU(2) x U(1)

Higgs

Chiral fermions

4D fermion effective Lagrangian in terms of mass eigenbasis

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty}\Bigg[\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} i\gamma^\mu\partial_\mu - m_n & 0 & 0 \\ 0 & i\gamma^\mu\partial_\mu - (m_n + m) & 0 \\ 0 & 0 & i\gamma^\mu\partial_\mu - (m_n - m) \end{pmatrix}\begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix}$$

$$+ \frac{g}{2}\left(\bar{\psi}_1^{(n)}, \bar{\tilde{\psi}}_2^{(n)}, \bar{\tilde{\psi}}_3^{(n)}\right)\begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & W_\mu^+ & W_\mu^+ \\ W_\mu^- & -\frac{W_\mu^3}{2} - \frac{B_\mu}{2\sqrt{3}} & -\frac{W_\mu^3}{2} + \frac{B_\mu}{2\sqrt{3}} \\ W_\mu^- & -\frac{W_\mu^3}{2} + \frac{B_\mu}{2\sqrt{3}} & -\frac{W_\mu^3}{2} - \frac{B_\mu}{2\sqrt{3}} \end{pmatrix}\gamma^\mu\begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix}\Bigg]$$

$$+ i\bar{t}_L\gamma^\mu\partial_\mu t_L + \bar{b}\left(i\gamma^\mu\partial_\mu - m\right)b + \frac{g}{\sqrt{2}}\left(\bar{t}\gamma^\mu L b W_\mu^+ + \bar{b}\gamma^\mu L t W_\mu^-\right) + \frac{g}{2}\left(\bar{t}\gamma^\mu L t + \bar{b}\gamma^\mu L b\right)W_\mu^3$$

$$+ \frac{\sqrt{3}g}{6}\left(\bar{t}\gamma^\mu L t + \bar{b}\gamma^\mu L b - 2\bar{b}\gamma^\mu R b\right)B_\mu$$

$$\begin{pmatrix} \tilde{\psi}_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix}$$

$$L \equiv \frac{1}{2}(1 - \gamma_5),\ R \equiv \frac{1}{2}(1 + \gamma_5),\ m_n = \frac{n}{R},\ g = \frac{g_5}{\sqrt{2\pi R}},\ m = \frac{1}{2}gv\,(= M_W)$$

Derivation of chiral suppression

4-Fermi operator

$$\sum_a \sum_{\alpha,\beta,\alpha',\beta'} \bar{s}_\alpha \left(\lambda^a\right)_{\alpha\beta} \gamma^\mu L d_\beta \bar{s}_{\alpha'} \left(\lambda^a\right)_{\alpha'\beta'} \gamma_\mu R d_{\beta'} = -\frac{1}{6} \bar{s}_\alpha \gamma^\mu L d_\alpha \bullet \bar{s}_\beta \gamma_\mu R d_\beta + \frac{1}{2} \bar{s}_\alpha \gamma^\mu L d_\beta \bullet \bar{s}_\beta \gamma_\mu R d_\alpha$$

$$\lambda^a_{ij} \lambda^a_{kl} = -\frac{1}{6} \delta_{ij} \delta_{kl} + \frac{1}{2} \delta_{il} \delta_{jk}$$

Hadronic matrix element

$$\left\langle \bar{K} \left| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^b_\alpha \bullet \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^{b'}_\alpha \right| K \right\rangle$$

$$= \sum_n \left[2 \left\langle \bar{K} \left| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^b_\alpha \right| n \right\rangle \left\langle n \left| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^{b'}_\alpha \right| K \right\rangle - 2 \left\langle \bar{K} \left| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^{b'}_\beta \right| n \right\rangle \left\langle n \left| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^b_\alpha \right| K \right\rangle \right]$$

$$\approx 2 \left\langle \bar{K} \left| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^b_\alpha \right| 0 \right\rangle \left\langle 0 \left| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^{b'}_\alpha \right| K \right\rangle - 2 \left\langle \bar{K} \left| \bar{s}^a_\alpha \left(\gamma^\mu L\right)_{ab} d^{b'}_\beta \right| 0 \right\rangle \left\langle 0 \left| \bar{s}^{a'}_\beta \left(\gamma_\mu R\right)_{a'b'} d^b_\alpha \right| K \right\rangle$$

$$= -\frac{1}{2} \left\langle \bar{K} \left| \bar{s}_\alpha \gamma^\mu \gamma^5 d_\alpha \right| 0 \right\rangle \left\langle 0 \left| \bar{s}_\beta \gamma_\mu \gamma^5 d_\beta \right| K \right\rangle - 4 \left\langle \bar{K} \left| \bar{s}^a_\alpha \left(R\right)_{ab'} d^{b'}_\beta \right| 0 \right\rangle \left\langle 0 \left| \bar{s}^{a'}_\beta \left(L\right)_{a'b} d^b_\alpha \right| K \right\rangle$$

"vacuum saturation"

$$= -\frac{1}{2} \left[-\frac{f_K^2}{2m_K} \left(p^\mu_K\right)^2 \right] + \frac{\delta_{ab}}{3} \frac{\delta_{ab}}{3} \left\langle \bar{K} \left| \bar{s}_\alpha \gamma^5 d_\alpha \right| 0 \right\rangle \left\langle 0 \left| \bar{s}_\beta \gamma^5 d_\beta \right| K \right\rangle$$

Fierz transformation

$$= \frac{1}{4} f_K^2 m_K + \frac{1}{6} \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K \approx \frac{1}{6} \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K$$

$$\langle 0 | j_5^\mu | K \rangle = \langle 0 | \bar{s}_\alpha \gamma^\mu \gamma^5 d_\alpha | K \rangle = \frac{f_K}{\sqrt{2m_K}} p_K^\mu, \langle 0 | \bar{s}_\alpha \gamma^\mu \gamma^5 d_\beta | K \rangle = \frac{1}{3} \delta_{\alpha\beta} \langle 0 | \bar{s}_\alpha \gamma^\mu \gamma^5 d_\alpha | K \rangle$$

$$\langle 0 | \bar{s}_\alpha \gamma^5 d_\alpha | K \rangle = -\frac{f_K}{\sqrt{2m_K}} \frac{m_K^2}{m_d + m_s}, \langle 0 | \bar{s}_\alpha \gamma^5 d_\beta | K \rangle = \frac{1}{3} \delta_{\alpha\beta} \langle 0 | \bar{s}_\alpha \gamma^5 d_\alpha | K \rangle$$

The relevant hadronic matrix elements are written by use of the "bag parameters" B_4, B_5, which denote the deviation from the approximation of vacuum saturation and whose numerical results are obtained by lattice calculations $B_4 = 0.81$, $B_5 = 0.56$ [18]:

$$\langle \bar{K} | \bar{s}_\alpha \gamma^\mu L d_\alpha \cdot \bar{s}_\beta \gamma_\mu R d_\beta | K \rangle \approx \frac{B_5}{6} \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K, \tag{4.9}$$

$$\langle \bar{K} | \bar{s}_\alpha \gamma^\mu L d_\beta \cdot \bar{s}_\beta \gamma_\mu R d_\alpha | K \rangle \approx \frac{B_4}{2} \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K, \tag{4.10}$$

$$\frac{1}{4} \langle \bar{K} | \bar{s} \lambda^a \gamma^\mu L d \cdot \bar{s} \lambda^a \gamma_\mu R d | K \rangle$$
$$= -\frac{1}{6} \langle \bar{K} | \bar{s}_{\alpha L} \gamma^\mu d_\alpha \cdot \bar{s}_{\beta R} \gamma_\mu d_{\beta R} | K \rangle + \frac{1}{2} \langle \bar{K} | \bar{s}_{\alpha L} \gamma^\mu d_{\beta L} \cdot \bar{s}_{\beta R} \gamma_\mu d_{\alpha R} | K \rangle$$
$$\approx \left(\frac{B_4}{4} - \frac{B_5}{36} \right) \left(\frac{m_K}{m_d + m_s} \right)^2 f_K^2 m_K .$$