Physics of Gauge-Higgs Unification

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References

"New Ideas on Electroweak Symmetry Breaking"
Christophe Grojean, CERN-PH-TH/2006-172

"Holographic Methods and Gauge-Higgs Unification
In Flat Extra Dimensions"
Marco Serone, arXiv: 0909.5619 [hep-ph]

"Lecture on Gauge-Higgs Unification
in extra dimensions"
Csaba Csa'ki
Talk slides in Ringberg Phenomenology Workshop

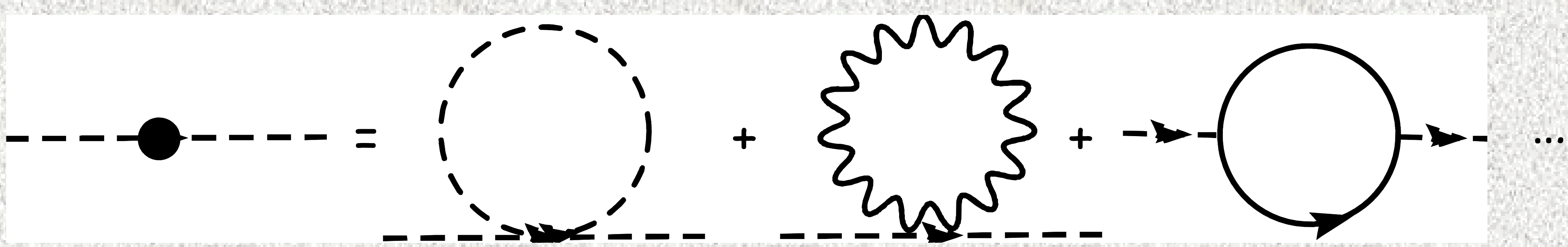
PLAN

- Introduction
- Higgs mass calculation
- Gauge-Higgs sector
- Yukawa structure
- Summary
- (Radius Stabilization)

Introduction

One of the problems in the Standard Model:
Hierarchy Problem

Quantum corrections to the Higgs mass
is sensitive to the cutoff scale of the theory



$$\delta m_H^2 \approx \frac{\Lambda^2}{16\pi^2}$$

Too large!!
(Natural cutoff scale is
Planck scale or GUT scale)

To get Higgs mass of weak scale,
an unnatural fine tuning of parameters are required

$$m_H^2 = m_0^2 + \delta m^2 \approx \mathcal{O}\left((100GeV)^2\right)$$

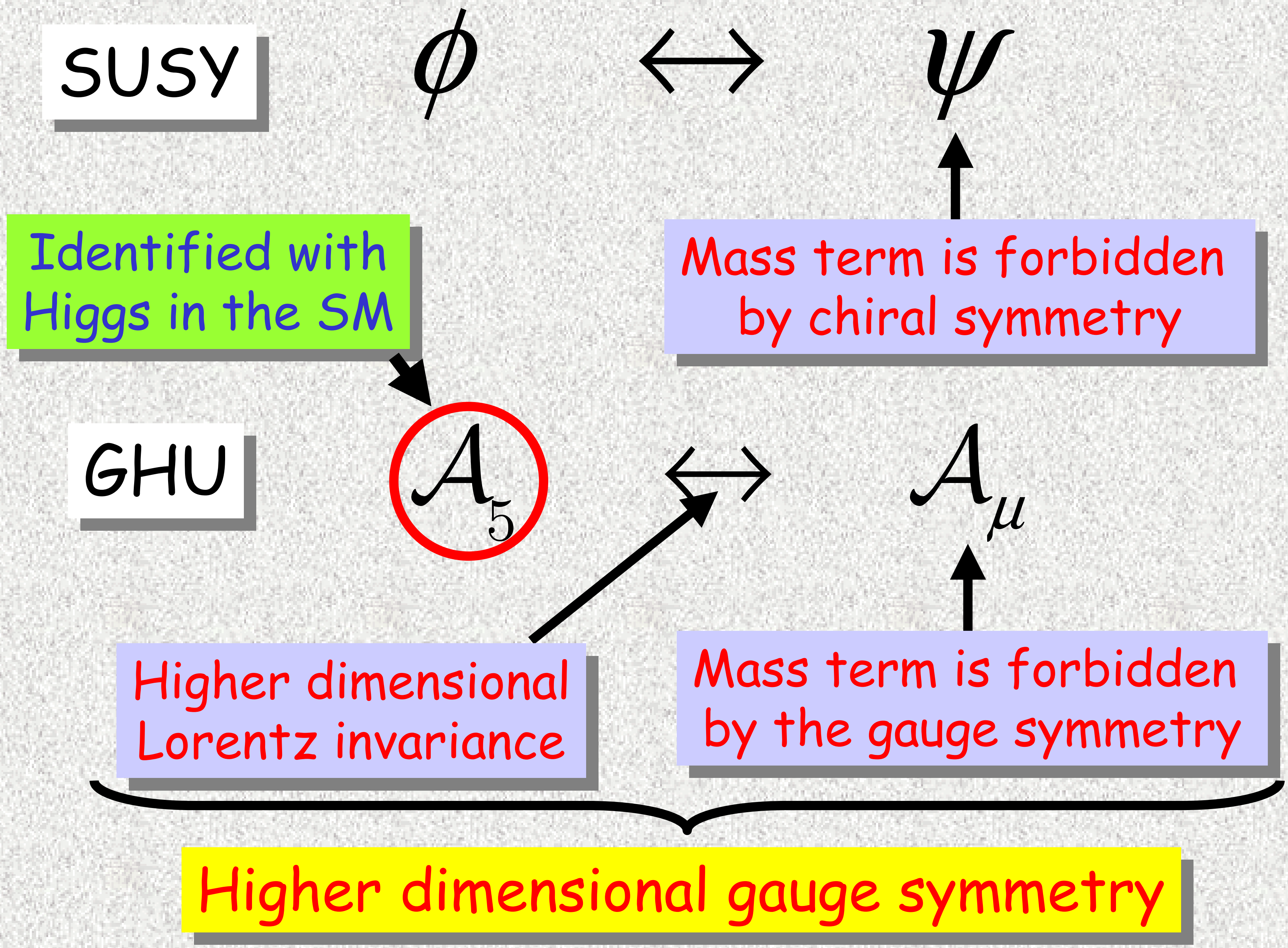
↑ classical ↑ Quantum corrections

Naively, we have $$m_0^2, \delta m^2 \approx \mathcal{O}\left((10^{18}GeV)^2\right)$$

32 powers of fine tuning

1.00000000000000000000000000000001 – 1 !!

Problem: We have NO symmetry forbidding the scalar mass



Indeed, the (local) mass term A_5^2 can be forbidden by the gauge symmetry for 5th component of the gauge field

$$\because A_5 \to A_5 + \partial_5 \varepsilon(x,y) + i\left[\varepsilon(x,y), A_5\right]$$

In other words, no local counter term is allowed

⇒ No quadratic divergence, finite

This symmetry is very useful in the orbifold model since it is operative even on the branes G → H

Gersdorff, Irges & Quiros (2002)

$$\because A_5 \to A_5 + \partial_5 \underbrace{\varepsilon_{G/H}(x,y)}_{Z_2 \text{ odd}} + i\left[\underbrace{\varepsilon_H(x,y)}_{Z_2 \text{ even}}, A_5\right]$$

No quadratic divergence from brane localized Higgs mass

Explicit calculations of Higgs mass

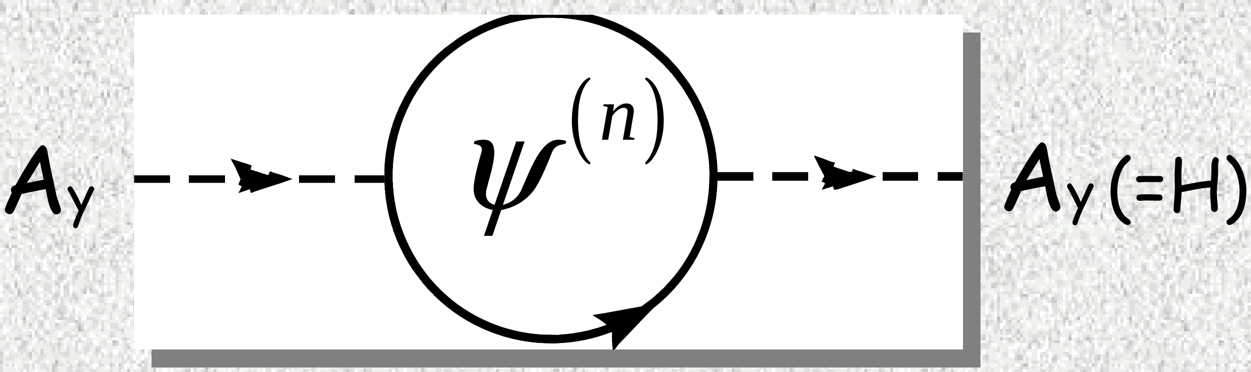
- D-dim QED on S^1@1-loop Hatanaka, Inami & Lim (1998)
- 5D Non-Abelian gauge theory on S^1/Z_2@1-loop
  Gersdorff, Irges & Quiros (2002)
- 6D Non-Abelian gauge theory on T^2@1-loop
  Antoniadis, Benakli & Quiros (2001)
- 6D Scalar QED on S^2@1-loop Lim, NM & Hasegawa (2006)
- 5D QED on S^1@2-loop
  NM & Yamashita (2006); Hosotani, NM, Takenaga & Yamashita (2007)
- 5D Gravity on S^1 (GGH) Hasegawa, Lim & NM (2004)

…

Higgs mass calculation

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



L=2πR

$$m_H^2 = ie_D^2 \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} Tr\left[\gamma_y \frac{1}{\not k - m}\gamma^y \frac{1}{\not k - m}\right]$$ (No sum)

$$\xrightarrow{L\to\infty} \frac{i}{D+1} e_{D+1}^2 \int \frac{d^{D+1}k}{(2\pi)^{D+1}} Tr\left[\gamma_M \frac{1}{\not k - m}\gamma^M \frac{1}{\not k - m}\right] (M = 0,1,\cdots D)$$

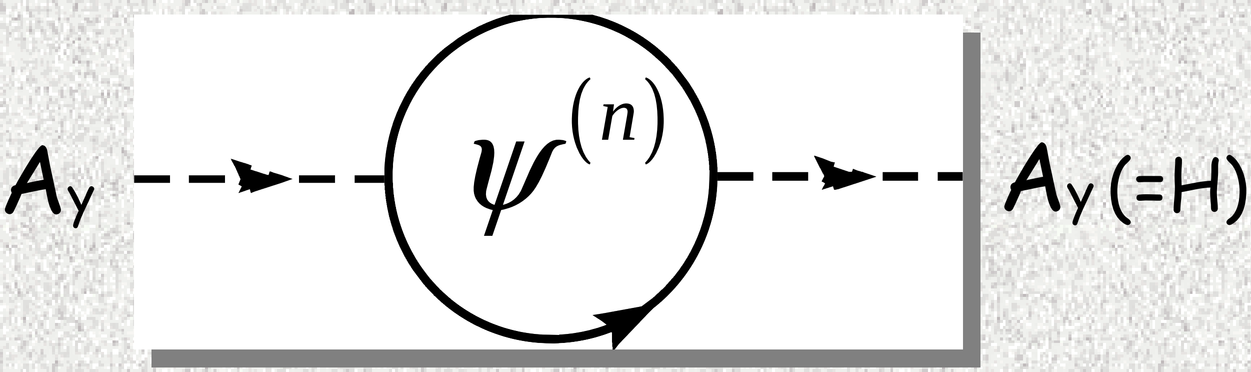
$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \int \frac{d^{D+1}k}{(2\pi)^{D+1}} \left[\frac{1-D}{k^2-m^2} - \frac{2m^2}{(k^2-m^2)^2}\right]$$

$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \left(1-D+2m^2\frac{\partial}{\partial m^2}\right) \int \frac{d^{D+1}k}{(2\pi)^{D+1}} \frac{1}{k^2-m^2}$$

$$= \frac{i}{D+1} e_{D+1}^2 2^{[(D+1)/2]} \frac{-i}{(4\pi)^{(D+1)/2}} \Gamma\left(\frac{1-D}{2}\right)\left(1-D+2m^2\frac{\partial}{\partial m^2}\right)(m^2)^{(D-1)/2} = 0$$

Consider (D+1)-dim QED on S^1

Hatanaka, Inami & Lim (1998)



Boundary condition

$$\psi(x_\mu, y+L) = e^{i\alpha}\psi(x_\mu, y)$$

$$m_H^2 = ie_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} \left[-\frac{1}{((2\pi n+\alpha)/L)^2 + \rho^2} + \frac{2\rho^2}{\left[((2\pi n+\alpha)/L)^2 + \rho^2\right]^2} \right]$$

$$= -ie_D^2 2^{[(D+1)/2]} \int \frac{d^D k}{(2\pi)^D} \left(1+\rho\frac{\partial}{\partial\rho}\right)\left(\frac{L}{2\rho}\right) \frac{\sinh(\rho L)}{\cosh(\rho L) - \cos\alpha}$$

L=2πR

$$\rho^2 = -k^2 + m^2$$

$$= \frac{e_D^2 L^2}{2^{D-[(D+1)/2]}\pi^{D/2}\Gamma(D/2)} \int_0^\infty dk\, k_E^{D-1} \frac{1-\cosh\left(\sqrt{k_E^2+m^2}L\right)\cos\alpha}{\left[\cosh\left(\sqrt{k_E^2+m^2}L\right) - \cos\alpha\right]^2} < \infty$$

Superconvergent!!

(Nonlocal mass: Wilson line phase $\alpha = g\oint dy A_y$)

Ex. take D=4 (5 dimension case) & m=0, $\alpha = \pi$

$$m_H^2 = \frac{e_4^2}{4\pi^2} \frac{1}{(2\pi R)^2} \int_0^\infty ds s^3 \frac{1 - \cosh s \cos\alpha}{[\cosh s - \cos\alpha]^2} \Bigg|_{\alpha=\pi}$$

$$= \frac{9e_4^2}{16\pi^4 R^2} \zeta(3) = \frac{9e_4^2}{4\pi^4} \underbrace{\zeta(3)}_{1.2} m_W^2$$

$m_W = \pi / R$

Higgs mass is too small

This is the generic prediction of gauge-Higgs unification

Way out to get Higgs mass > 114 GeV

1: Realizing small Higgs VEV $\alpha \ll 1$
by choosing appropriate matter content

$$m_H \sim m_W/(4\pi\alpha) \quad (m_W = \alpha/R)$$

Haba, Hosotani, Kawamura & Yamashita etc

2: D > 5 dimensions

F_{ij}^2 contains the Higgs quartic coupling g^2[A_i, A_j]^2 in general. Higgs mass is generated at tree level
$m_H = 2m_W$ is predicted in 6D on T^2/Z_3 model

Scrucca, Serone, Silvestrini & Wulzer (2003)

3: Warped dimension (ex. Randall-Sundrum model)

Higgs mass is enhanced by curvature scale $k\pi R \sim 30$

Contino, Nomura & Pomarol (2003)

Gauge-Higgs sector

Model building of the gauge-Higgs unification

A_5 is an SU(2) adjoint as it stands, not SU(2) doublet
⇒ need to enlarge the gauge group

$G \to SU(2)_L \times U(1)_Y$
adj → doublet + other reps

➡ Simplest G
SU(3)

Consider 5D SU(3) model on S^1/Z_2 with Parity: P = diag (-,-,+)

$$PA_\mu(x, y_i - y)P^\dagger = A_\mu(x, y_i + y),\ PA_5(x, y_i - y)P^\dagger = -A_5(x, y_i + y)$$

$$\Rightarrow\quad A_\mu = \begin{pmatrix} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ (-,-) & (-,-) & (+,+) \end{pmatrix},\ A_5 = \begin{pmatrix} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ (+,+) & (+,+) & (-,-) \end{pmatrix}$$

(+,+) mode has only massless mode ("0 mode")

$$A_\mu^{(0)} = \frac{1}{2}\begin{pmatrix} W_\mu^3 + B_\mu^3/\sqrt{3} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 & 0 \\ 0 & 0 & -2B_\mu/\sqrt{3} \end{pmatrix}, A_5^{(0)} = \frac{1}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}$$

SU(2) x U(1) gauge fields — Higgs doublets

Using mode expansions

$$A_M^{(+,+)}(x,y) = \frac{1}{\sqrt{2\pi R}}\left[A_M^{(0)}(x) + \sqrt{2}\sum_{n=1}^{\infty} A_M^{(n)}(x)\cos\left(\frac{n}{R}y\right)\right]$$

$$A_M^{(-,-)}(x,y) = \frac{1}{\sqrt{\pi R}}\sum_{n=1}^{\infty} A_M^{(n)}(x)\sin\left(\frac{n}{R}y\right)$$

Gauge boson spectrum

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{a}{g_5 R}$$

$$M_{W_n} = \frac{n+a}{R}, M_{Z_n} = \frac{n+2a}{R}, M_{\gamma_n} = \frac{n}{R}, \left\langle A_5^{(0)} \right\rangle = \frac{a}{g_5 R}$$

- W, Z, γ are identified with zero modes:
  $M_{W_0} = a/R$, $M_{Z_0} = 2a/R$, $M_{\gamma_0} = 0$
- $M_Z = 2M_W \to \cos\theta_W = \frac{1}{2}$ ($\sin^2\theta_W = \frac{3}{4}$ >> 0.23)
- The spectrum is invariant under an integer shift of "a" & a ⇔ −a → physical range [0, 1/2]
  (this type of spectrum is specific to GHU compared to UED case: $M_{W_n} = \sqrt{M_W^2 + (n/R)^2}$)
- Non-zero KK modes of A_5 are eaten by non-zero KK modes of A_μ (Higgs mechanism)

Hypercharge of the doublet

Check the hypercharge of Higgs doublet

$$\delta_{U(1)} A_5^{(0)} = g\left[T^8, A_5^{(0)}\right] = \frac{g}{2\sqrt{3}}\left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ H^- & H^{0*} & 0 \end{pmatrix}\right]$$

$$= \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -2H^- & -2H^{0*} & 0 \end{pmatrix} - \frac{g}{2\sqrt{3}}\begin{pmatrix} 0 & 0 & -2H^+ \\ 0 & 0 & -2H^0 \\ H^- & H^{0*} & 0 \end{pmatrix} = \frac{g\sqrt{3}}{2}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H^0 \\ -H^- & -H^{0*} & 0 \end{pmatrix}$$

$$\Rightarrow \quad \sin^2\theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{\left(\sqrt{3}g\right)^2}{g^2 + \left(\sqrt{3}g\right)^2} = \frac{3}{4} \neq 0.23(\text{Exp})$$

Too Big!!

Not new: Fairlie, Manton (6D on S2 with monopole background)

	G2	SO(5)	SU(3)
$\sin^2\Theta_W$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$

Way out to get a correct Weinberg angle

1: Additional U(1) SU(3) x U(1)′ →SU(2)L x U(1)Y x U(1)X

Scrucca, Serone & Silvestrini (2003)

$$A_Y = \frac{g' A_8 + \sqrt{3} g A'}{\sqrt{3g^2 + g'^2}}, A_X = \frac{\sqrt{3} g A_8 - g' A'}{\sqrt{3g^2 + g'^2}} \Rightarrow g_Y = \frac{\sqrt{3} g g'}{\sqrt{3g^2 + g'^2}}$$

$$\Rightarrow \quad \sin^2 \theta_W = \frac{g_Y^2}{g^2 + g_Y^2} = \frac{3}{4 + 3g^2 / g'^2}$$

2: Localized gauge kinetic terms

$$\mathcal{L} = -\frac{1}{2g_5^2} TrF_{MN} F^{MN} - \left[\frac{1}{2g_4^2} \delta(y) + \frac{1}{2g_4'^2} \delta(y - \pi R) \right] TrF_{\mu\nu} F^{\mu\nu}$$

SU(3) invariant SU(2) x U(1) invariant

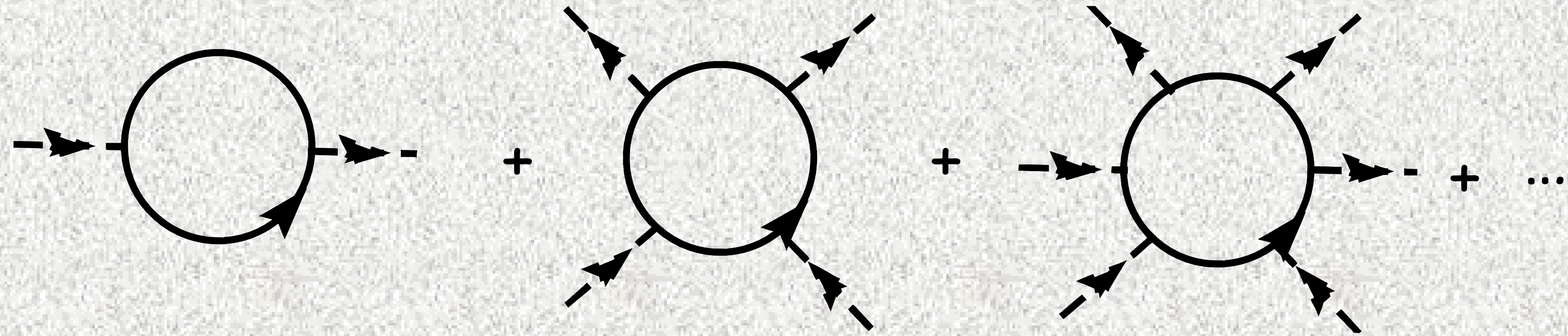
Electroweak symmetry breaking

In the gauge-Higgs unification, electroweak symmetry breaking is dynamically realized by the Hosotani mechanism

Hosotani (1983,1989)

Higgs potential is radiatively generated since the tree level potential is forbidden by the gauge invariance (Coleman-Weinberg potential)

$V(A_5) =$



$$V(a) = (-1)^F \frac{(\mathrm{DOF})}{2} \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{2\pi R} \sum_n \log\left(p_E^2 + m_n^2\right)$$

KK mass

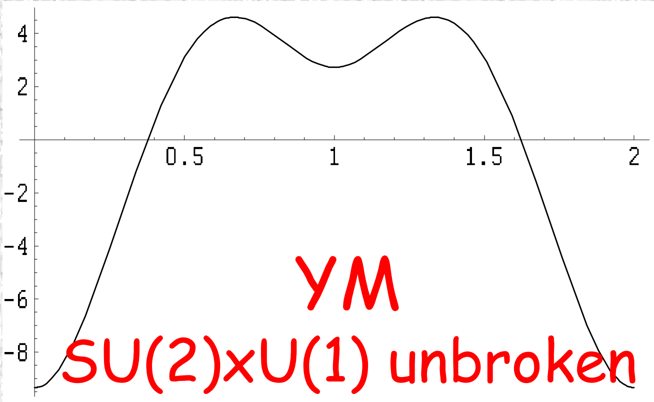
Ex. 5D SU(3) model on S^1/Z2 with N_f fundamental & N_a adjoint fermions

Kubo, Lim & Yamashita (2002)

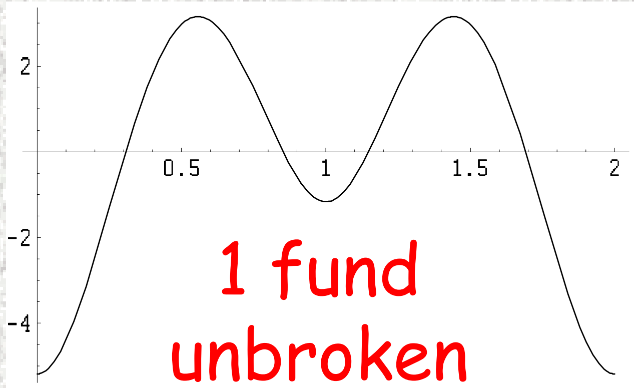
$$V(a) = \frac{3}{128\pi^7 R^5} \sum_{n=1}^{\infty} \frac{1}{n^5} \Big[(4N_a - 3) \underbrace{\big(\cos[2\pi n a] + 2\cos[\pi n a]\big)}_{\text{adjoint}} + 4N_f \underbrace{\cos[\pi n a]}_{\text{fund}} \Big]$$

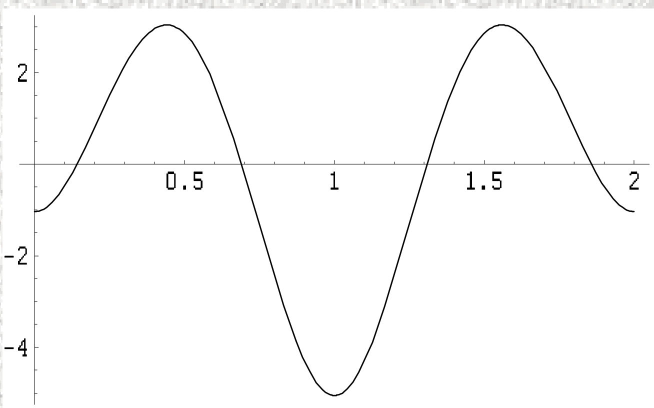
Gauge + ghost

V(a)



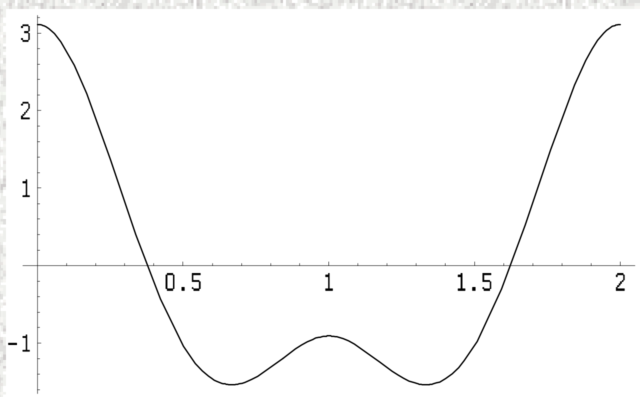
a





2 fund

SU(2)xU(1) -> U(1)xU(1)



1 adj

SU(2)xU(1) -> U(1)

Wilson line phase

$$W = \mathcal{P}\exp\left(ig\oint_{S^1} dy A_5\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i\sin(\pi a) \\ 0 & i\sin(\pi a) & \cos(\pi a) \end{pmatrix} \quad (a \bmod 2) = \begin{cases} SU(2)\times U(1) \text{ for a = 0} \\ U(1)'\times U(1) \text{ for a = 1} \\ U(1)_{em} \text{ for other cases} \end{cases}$$

$$\langle A_5 \rangle = \frac{a}{gR}\frac{T^6}{2} \equiv A_5^{6(0)}\frac{T^6}{2}$$

$$T^3 = diag(1,-1,0)$$

$$T^8 = diag(1,1,-2)/\sqrt{3}$$

$$a = 1 : W = diag(1,-1,-1) \Rightarrow [W, T^3] = [W, T^8] = 0$$

U(1) x U(1)' unbroken

$$0 < a < 1 : [W, \sqrt{3}T^3 + T^8] = [W, \sin\theta_W T^3 + \cos\theta_W T^8] = 0$$

$U(1)_{em}$ unbroken

Yukawa Structure

Quark & Lepton embedding

Consider a fundamental rep of SU(3)

$\mathbf{3}$ = (q, q-1, 1-2q)^T (q: electric charge)

Putting q=2/3, we get (+, +, −)L

$\mathbf{3} = \mathbf{2}_{1/6} + \mathbf{1}_{-1/3}$ = (2/3, -1/3, -1/3)^T = (u_L, d_L, d_R)^T

Only fundamental reps cannot incorporate right-handed up-type quarks as well as leptons

2-rank sym: $\mathbf{6}^* = \begin{cases} \mathbf{3}_{L-1/3} + \mathbf{2}_{L1/6}\,(Q) + \mathbf{1}_{L2/3} \\ \mathbf{3}_{R-1/3} + \mathbf{2}_{R1/6} + \mathbf{1}_{R2/3}\,(u_R) \end{cases}$

3-rank sym: $\mathbf{10} = \begin{cases} \mathbf{4}_{L1/2} + \mathbf{3}_{L0} + \mathbf{2}_{L-1/2}\,(L) + \mathbf{1}_{L-1} \\ \mathbf{4}_{R1/2} + \mathbf{3}_{R0} + \mathbf{2}_{R-1/2} + \mathbf{1}_{R-1}\,(e_R) \end{cases}$

Many massless exotics ⇒ brane localized mass term

Big Hurdle

In the gauge-Higgs unification,

Yukawa coupling = gauge coupling

How can we get fermion mass hierarchy???

As will be shown below,
fermion masses except for top quark are relatively easy

1: Localizing fermions@different point in 5^{th} direction

Yukawa ~ exponentially suppressed overlap integral

Arkani-Hamed & Schmaltz (1999)

2: Bulk fermions mixed with localized fermions @the fixed points

Non-local Yukawa coupling

Csaki, Grojean & Murayama (2002)

1: Yukawa coupling from localizing fermions @different points

1: To localize fermions at different points along the 5^{th} direction, bulk masses are introduced

2: To be consistent with Z_2 orbifold, Z_2 parity of bulk mass must be odd ⇒ kink mass

Consider a 5D fermion satisfying the following Dirac equation

$$0=\left[i\Gamma^M D_M - M\varepsilon(y)\right]\psi(x,y)$$

$$D_M=\partial_M - igA_M,\Gamma^M=\left(\gamma^\mu,i\gamma^5\right),(M=0,1,2,3,5),\varepsilon(y)=\begin{cases}1\,(y>0)\\-1\,(y<0)\end{cases}$$

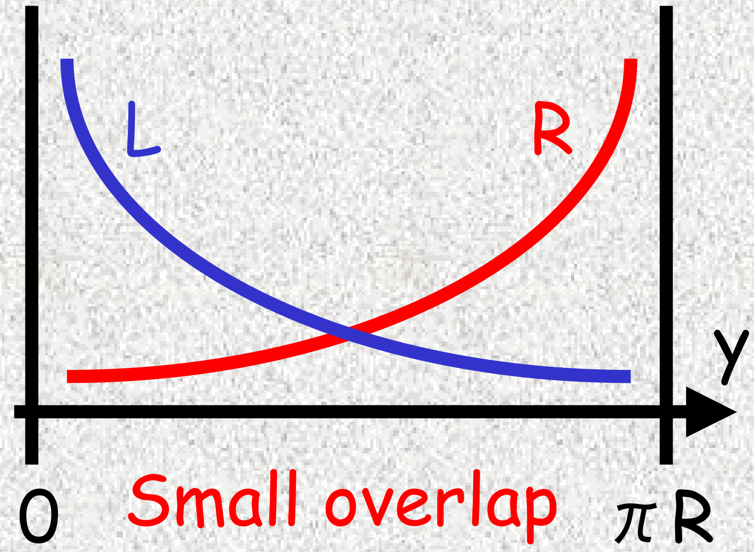
Focusing zero modes

$$\psi(x,y)\sim\psi_{L(R)}^{(0)}(x)\,f_{L(R)}^{(0)}(y),\ \gamma^5\psi_{L(R)}=(-)\psi_{L(R)}$$

Zero mode wave functions

$$0=\left[\partial_y+M\varepsilon(y)\right]f_L^{(0)}(y)\rightarrow f_L^{(0)}(y)=\sqrt{\frac{M}{1-e^{-2\pi MR}}}e^{-M|y|}$$

$$0=\left[\partial_y-M\varepsilon(y)\right]f_R^{(0)}(y)\rightarrow f_R^{(0)}(y)=\sqrt{\frac{M}{e^{2\pi MR}-1}}e^{M|y|}$$



4D effective Yukawa coupling

$$Y=g_4\int_{-\pi R}^{\pi R}dy f_L^{(0)}(y)\,f_R^{(0)}(y)=g_4\int_{-\pi R}^{\pi R}dy\sqrt{\frac{M^2}{\left(1-e^{-2\pi MR}\right)\left(e^{2\pi MR}-1\right)}}$$

$$\approx 2\pi MRg_4e^{-\pi MR}\leq g_4\Leftrightarrow m_f\leq m_W$$

$\pi MR >> 1$

Fermion masses except top is easy, but top is hard

No need of unnatural fine-tuning for 5D parameters M,R

2: Mixing between bulk and boundary localized fermions

Csaki, Grojean & Murayama (2002)

Consider the massive bulk fermion coupling with SM fermions on the branes

$$\mathcal{L}_{Bulk} = \bar{\Psi}(x,y)\left(i\Gamma^M D_M - M(\varepsilon)\right)\Psi(x,y), \Psi = \left(\psi^d, \chi^s\right)^T$$

$$\mathcal{L}_{Brane} = \delta(y-y_L)\left[i\bar{Q}_L\bar{\sigma}^\mu\partial_\mu Q_L + \frac{\varepsilon_L}{\sqrt{\pi R}}\bar{\psi}^d Q_L + h.c.\right] + \delta(y-y_R)\left[i\bar{q}_L\bar{\sigma}^\mu\partial_\mu q_L + \frac{\varepsilon_R}{\sqrt{\pi R}}\bar{q}_R\chi^s + h.c.\right]$$

Mixing mass term between bulk & brane fermions

Integrating out massive fermion generates mass term as

$$\varepsilon_L\varepsilon_R\pi MRe^{-\pi MR}\bar{q}_R e^{ig\int_0^{\pi R}dyA_y}Q_L \Rightarrow m_f \propto \varepsilon_L\varepsilon_R\pi MRe^{-\pi MR}M_W$$

Exponentially suppressed coupling

⇒ easy to generate fermion masses except for top

How do we obtain top mass???

Top mass generation

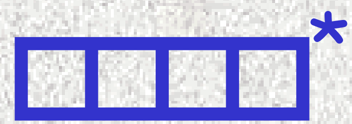
Cacciapaglia, Csaki & Park (2005)

Consider large dimensional reps,
then an upper bound on fermion mass is modified as follows

$$m_t \leq \sqrt{n}\, m_W$$

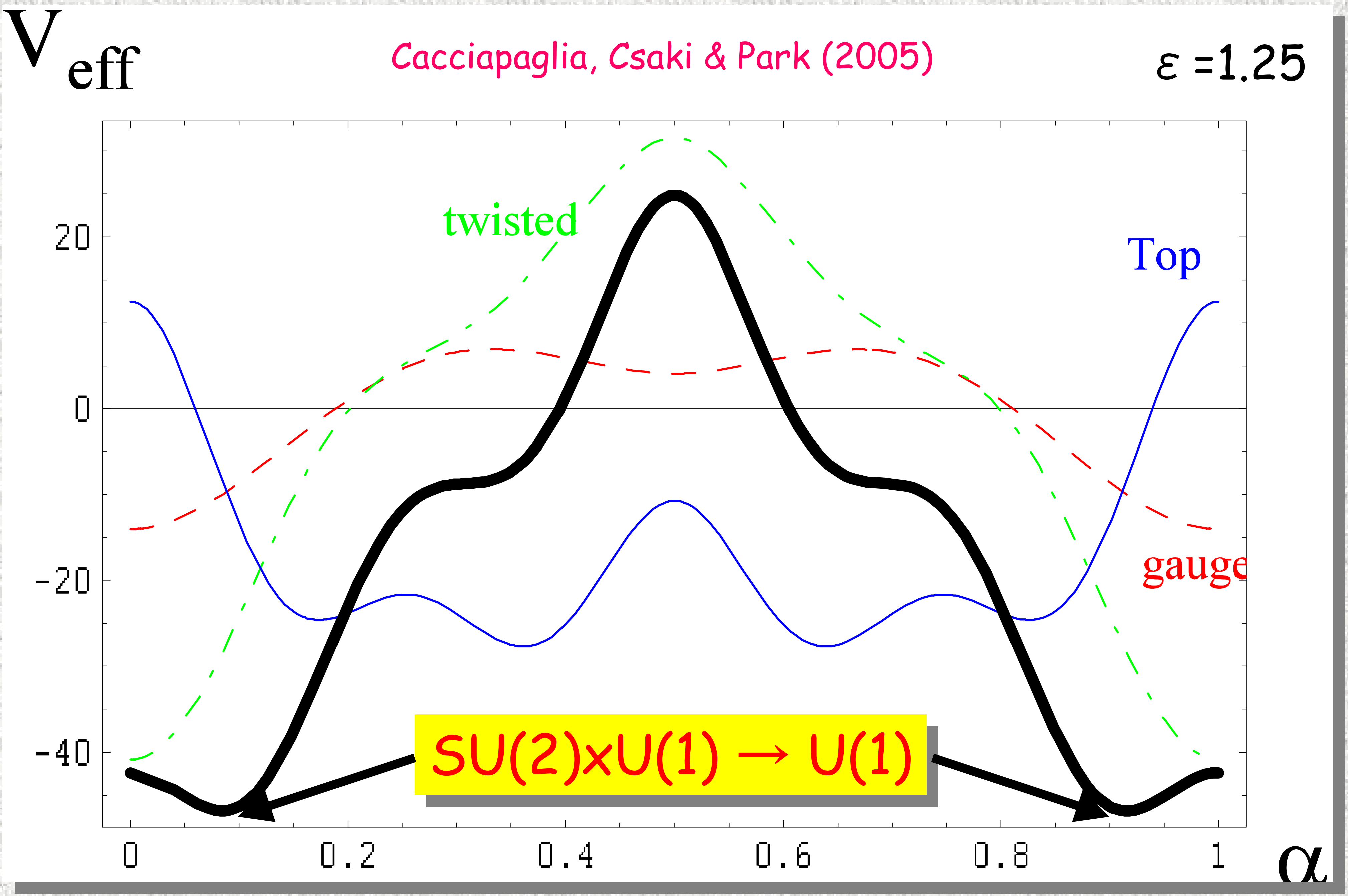
(n: # of indices of rep)

For $m_t = 2m_W$ ⇒ need a **4-index** rep top is embedded
To saturate this bound, bulk mass should be zero

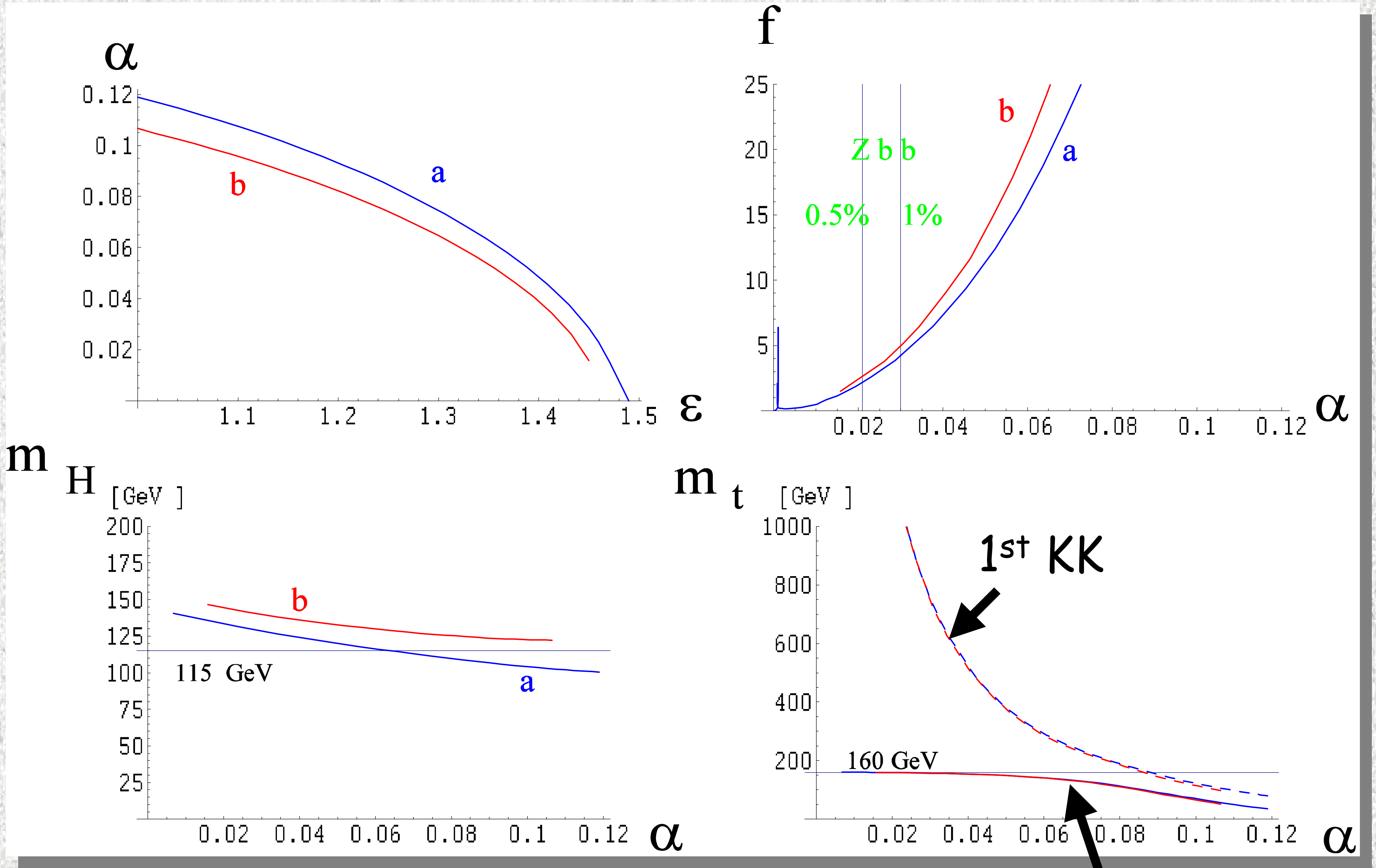
Simplest example: 

$(\mathbf{15^*})_{-2/3} \longrightarrow (\mathbf{1}, 2/3)(t_R) + (\mathbf{2}, 1/6)(t_L)$
$+ (\mathbf{3}, -1/3) + (\mathbf{4}, -5/6) + (\mathbf{5}, -4/3)$

Higgs potential (top (15*) + bottom (3) + tau (10))



Higgs mass, top mass,...etc



Model a: b(3), τ (10)   Model b: b(6), τ (3)

Sample points

α	$1/R$	f	m_H	m_t	m'_t
0.08	1 TeV	31% 42%	110 125 GeV	113 110 GeV	189 186 GeV
0.05	1.6 TeV	11% 14%	120 133 GeV	149 149 GeV	381 375 GeV
0.04	2 TeV	7% 9%	124 136 GeV	154 154 GeV	519 514 Gev
0.03	2.7 TeV	4% 5%	128 140 GeV	157 157 GeV	753 746 Gev
0.02	4 TeV	2% 2%	134 144 GeV	159 159 GeV	1224 1213 Gev

↑

Fine-tuning required to obtain the potential minimum

Model A (top low)
Model B (bottom low)

Summary

- Gauge-Higgs unification is a very attractive scenario beyond the Standard Model

- Controlled by gauge principle & very predictive
  Higgs mass, potential → finite

- Hard to generate flavor structure, but possible

- Today, we focus on the flat space case
  Warped space case is a hopeful direction
  In AdS/CFT correspondence,
  GHU in warped space ⇔ Little Higgs model

Various aspects of GHU have been studied so far

- S & T parameters (w/ Lim)
- g-2 (w/ Adachi & Lim)
- GUT extension (w/ Lim)
- Collider signature (w/ Okada)
- Finite temperature (w/ Takenaga)
- CP violation (w/ Adachi & Lim, Lim & Nishiwaki)
- Radius stabilization (w/ Sakamura)
- Flavor violation (w/ Adachi, Kurahashi & Lim)

Radius Stabilization in GHU

"Modulus Stabilization and IR-Brane Kinetic terms in Gauge-Higgs Unification"

N.M. & Y.Sakamura

JHEP1004 (2010) 100

(arXiv:1002.4259 [hep-ph])

Introduction

In gauge-Higgs unification models,
Higgs mass is controlled by the compactification scale as

$$m_H^2 \approx \frac{\alpha}{4\pi}\frac{1}{R^2}(\text{flat}), \frac{\alpha}{4\pi}\pi R k\left(k\pi e^{-\pi R k}\right)^2(\text{warped})$$

To get Higgs mass of order 100GeV,
we need 1/R ~O(TeV) or $k\pi R$~O(10)
Why???

Also from other phenomenological reasons,
such as the deviation from Newton law,
collider experiments etc,
Radion should be massive in some extent

We must therefore stabilize the radion somehow

Limits on Mass of Radion

This section includes limits on mass of radion, usually in context of Randall-Sundrum models. See the "Extra Dimension Review" for discussion of model dependence.

VALUE (GeV)	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	[72] ABBIENDI	05	OPAL	$e^+e^- \rightarrow Z$ radion
$\gtrsim 35$	[73] MAHANTA	00		$Z \rightarrow$ radion $\ell\overline{\ell}$
>120	[74] MAHANTA	00B		$p\overline{p} \rightarrow$ radion $\rightarrow \gamma\gamma$

[72] ABBIENDI 05 use e^+e^- collisions at $\sqrt{s} = 91$ GeV and $\sqrt{s} = 189$–209 GeV to place bounds on the radion mass in the RS model. See their Fig. 5 for bounds that depend on the radion-Higgs mixing parameter ξ and on $\Lambda_W = \Lambda_\phi/\sqrt{6}$. No parameter-independent bound is obtained.

[73] MAHANTA 00 obtain bound on radion mass in the RS model. Bound is from Higgs boson search at LEP I.

[74] MAHANTA 00B uses $p\overline{p}$ collisions at $\sqrt{s}= 1.8$ TeV; production via gluon-gluon fusion. Authors assume a radion vacuum expectation value of 1 TeV.

from Particle Data Group

Stabilization by a Bulk Scalar field

Goldberger & Wise, PRL83 (1999) 4922

- Radius is stabilized at tree level
- An additional scalar field needs to be introduced

Stabilization by Casimir energy

Garriga, Pujolas, Tanaka, NPB605 (2001) 192;
Goldberger & Rothstein, PLB491 (2000) 339;
Garriga & Pomarol, PLB560 (2003) 91...

- Radius is stabilized at quantum level
- Bulk gauge field and fermions are enough for the stabilization

No need to introduce extra fields!!

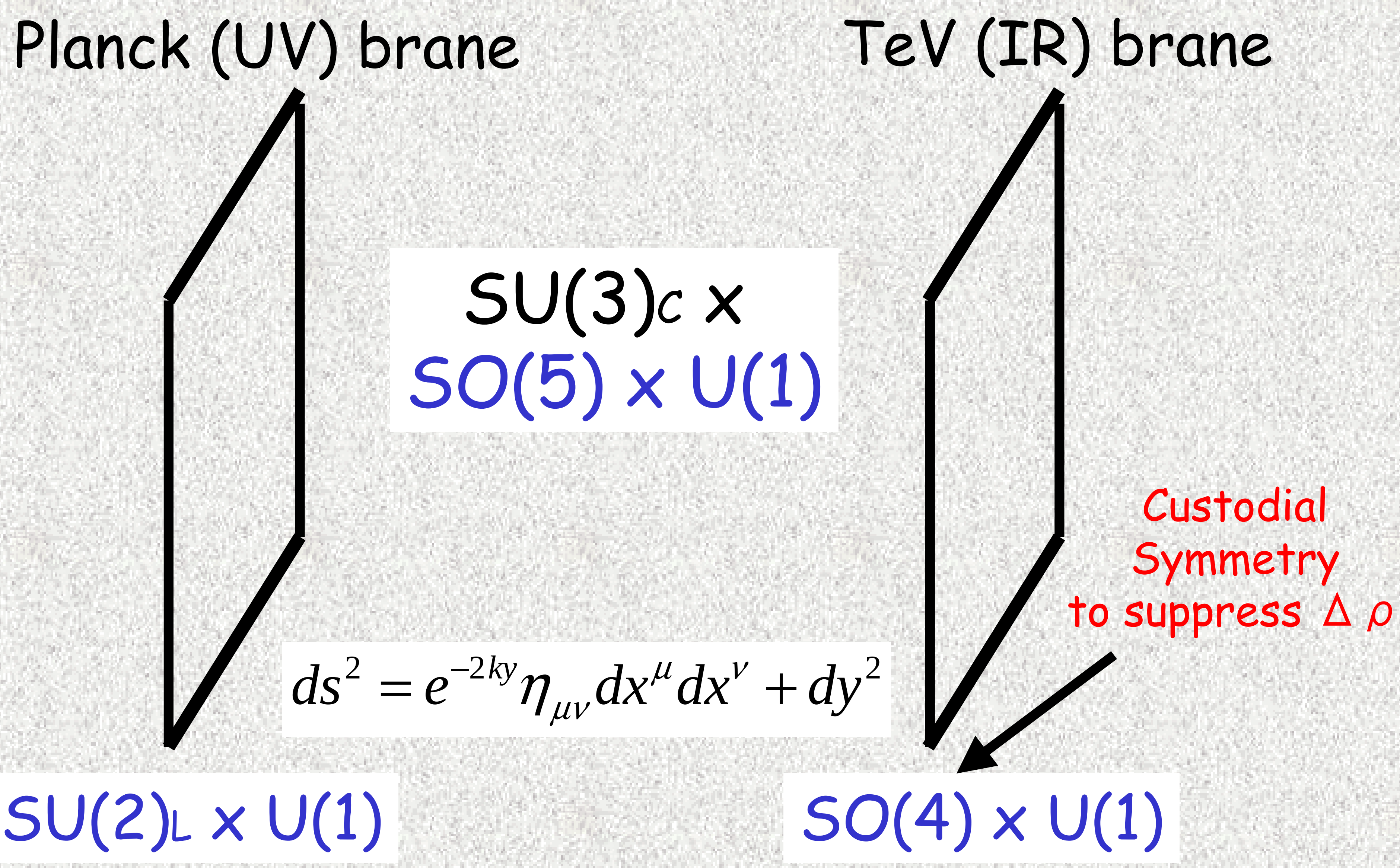
It is desirable to have a model
where the electroweak symmetry breaking &
the radius stabilization work simultaneously

⇓

We will find that
an SO(5)xU(1)x gauge-Higgs model on RS background
does these jobs successfully

SO(5) x U(1)x model on S1/Z2 in warped space

Agashe, Contino & Pomarol, NPB719 (2005) 165



Field content

Hosotani, Oda, Ohnuma & Sakamura, PRD78 (2008) 096002

Gauge field

G_μ : $SU(3)_C$ gauge field

A_μ : SO(5) gauge field

B_μ : U(1) gauge field

Spinor field (quark sector of 3rd generation)

Ψ_1, Ψ_2: SO(5) vector rep.

Color triplets

Components of Spinors

$$SO(5) \supset SO(4) = SU(2)_L \times SU(2)_R$$

Parity: P = diag$(-, -, -, -, +)$

$$\Psi_1 = \left[\begin{pmatrix} T \\ B \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, t' \right], \quad \Psi_2 = \left[\begin{pmatrix} U \\ D \end{pmatrix}, \begin{pmatrix} X \\ Y \end{pmatrix}, b' \right]$$

Qx = 2/3: B, t, t′, U → top quark

Qx = -1/3: D, b, b′, X → down quark

Exotic massless fermions from T, B, U, D, X, Y are removed by introducing brane fermions & mass terms

Brane localized kinetic terms

IR-brane kinetic terms of gauge fields are introduced,
which is required to stabilize the radius
as discussed by Garriga & Pomarol

$$\mathcal{L}_{IR}^{kin} = 2\sqrt{-g}\left[-\frac{\kappa_c}{4k} tr\left(F_{\mu\nu}^{(G)} F^{(G)\mu\nu}\right) - \frac{\kappa_w}{4k} tr\left(F_{\mu\nu}^{(A)} F^{(A)\mu\nu}\right) - \frac{\kappa_x}{4k} tr\left(F_{\mu\nu}^{(B)} F^{(B)\mu\nu}\right)\right]\delta(y-L)$$

- These terms will be generically induced by loop effects of bulk fields even if we do not have such terms at tree level
- Change the boundary conditions → repel the mode function away from IR brane
- We do not consider kinetic terms on UV brane or brane kinetic terms for fermions

Radion-Higgs potential

$$V\left(kL,\theta_H\right)=\frac{k^4}{16\pi^2}\left[\tau_{UV}+\tau_{IR}e^{-4kL}+e^{-4kL}\int_0^\infty dw\,w^3 v_{eff}\left(w;kL,\theta_H\right)\right]$$

$$v_{eff}\left(w;kL,\theta_H\right)=24\ln\left[1-\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_c}\left(w\right)}{K_0\left(we^{-kL}\right)I_0^{\kappa_c}\left(w\right)}\right]+9\ln\left[1-\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_w}\left(w\right)}{K_0\left(we^{-kL}\right)I_0^{\kappa_w}\left(w\right)}\right]$$

$$+3\ln\left[1-c_\phi^2\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_x}\left(w\right)}{K_0\left(we^{-kL}\right)I_0^{\kappa_x}\left(w\right)}-s_\phi^2\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_w}\left(w\right)}{K_0\left(we^{-kL}\right)I_0^{\kappa_w}\left(w\right)}\right]$$

$$-24\ln\left[1-\frac{I_{c_t-1/2}\left(we^{-kL}\right)K_{c_t-1/2}\left(w\right)}{K_{c_t-1/2}\left(we^{-kL}\right)I_{c_t-1/2}\left(w\right)}\right]+6\ln\left[1+\frac{e^{kL}\sin^2\theta_H}{2w^2\hat{F}_{0,0}^{\kappa_w}\hat{F}_{1,1}^{0}}\right]$$

$$+3\ln\left[1+\frac{e^{kL}\left(c_\phi^2\hat{F}_{0,0}^{\kappa_x}\hat{F}_{1,0}^{\kappa_w}+2s_\phi^2\hat{F}_{1,0}^{\kappa_x}\hat{F}_{1,0}^{\kappa_w}\right)\sin^2\theta_H}{2w^2\left(c_\phi^2\hat{F}_{0,0}^{\kappa_x}\hat{F}_{1,0}^{\kappa_w}+s_\phi^2\hat{F}_{1,0}^{\kappa_x}\hat{F}_{0,0}^{\kappa_w}\right)\hat{F}_{0,0}^{\kappa_w}\hat{F}_{1,1}^{0}}\right]$$

$$-12\ln\left[1+\frac{e^{kL}\sin^2\theta_H}{2w^2\hat{F}_{c_t+1/2,c_t+1/2}^{0}\hat{F}_{c_t-1/2,c_t-1/2}^{0}}\right]$$

Radion-Higgs potential

$$V\left(kL,\theta_H\right)=\frac{k^4}{16\pi^2}\left[\tau_{UV}+\tau_{IR}e^{-4kL}+e^{-4kL}\int_0^\infty dww^3 v_{eff}\left(w;kL,\theta_H\right)\right]$$

Tension @branes

W & Z

$$v_{eff}\left(w;kL,\theta_H\right)=24\ln\left[1-\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_c}(w)}{K_0\left(we^{-kL}\right)I_0^{\kappa_c}(w)}\right]+9\ln\left[1-\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_w}(w)}{K_0\left(we^{-kL}\right)I_0^{\kappa_w}(w)}\right]$$

gluon

$$+3\ln\left[1-c_\phi^2\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_x}(w)}{K_0\left(we^{-kL}\right)I_0^{\kappa_x}(w)}-s_\phi^2\frac{I_0\left(we^{-kL}\right)K_0^{\kappa_w}(w)}{K_0\left(we^{-kL}\right)I_0^{\kappa_w}(w)}\right]$$

γ

t & b

$$-24\ln\left[1-\frac{I_{c_t-1/2}\left(we^{-kL}\right)K_{c_t-1/2}(w)}{K_{c_t-1/2}\left(we^{-kL}\right)I_{c_t-1/2}(w)}\right]+6\ln\left[1+\frac{e^{kL}\sin^2\theta_H}{2w^2\hat{F}_{0,0}^{\kappa_w}\hat{F}_{1,1}^{0}}\right]$$

W

Z

$$+3\ln\left[1+\frac{e^{kL}\left(c_\phi^2\hat{F}_{0,0}^{\kappa_x}\hat{F}_{1,0}^{\kappa_w}+2s_\phi^2\hat{F}_{1,0}^{\kappa_x}\hat{F}_{1,0}^{\kappa_w}\right)\sin^2\theta_H}{2w^2\left(c_\phi^2\hat{F}_{0,0}^{\kappa_x}\hat{F}_{1,0}^{\kappa_w}+s_\phi^2\hat{F}_{1,0}^{\kappa_x}\hat{F}_{0,0}^{\kappa_w}\right)\hat{F}_{0,0}^{\kappa_w}\hat{F}_{1,1}^{0}}\right]$$

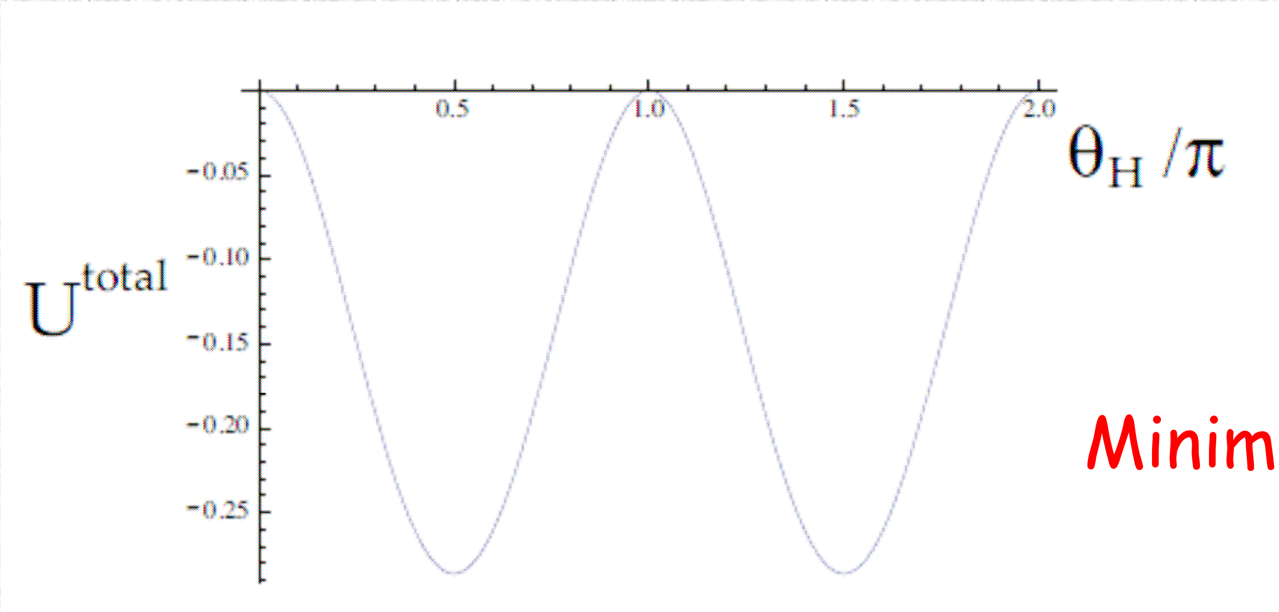
$$-12\ln\left[1+\frac{e^{kL}\sin^2\theta_H}{2w^2\hat{F}_{c_t+1/2,c_t+1/2}^{0}\hat{F}_{c_t-1/2,c_t-1/2}^{0}}\right]$$

t & b

θ_H dependent potential

$$v_{eff}(w;kL,\theta_H) \supset 6\ln\left[1+\frac{e^{kL}\sin^2\theta_H}{2w^2\hat{F}^{\kappa_w}_{0,0}\hat{F}^{0}_{1,1}}\right] - 12\ln\left[1+\frac{e^{kL}\sin^2\theta_H}{2x^2\hat{F}^{0}_{c_t+1/2,c_t+1/2}\hat{F}^{0}_{c_t-1/2,c_t-1/2}}\right]$$
$$+3\ln\left[1+\frac{e^{kL}\left(c_\phi^2\hat{F}^{\kappa_x}_{0,0}\hat{F}^{\kappa_w}_{1,0}+2s_\phi^2\hat{F}^{\kappa_x}_{1,0}\hat{F}^{\kappa_w}_{1,0}\right)\sin^2\theta_H}{2x^2\left(c_\phi^2\hat{F}^{\kappa_x}_{0,0}\hat{F}^{\kappa_w}_{1,0}+s_\phi^2\hat{F}^{\kappa_x}_{1,0}\hat{F}^{\kappa_w}_{0,0}\right)\hat{F}^{\kappa_w}_{0,0}\hat{F}^{0}_{1,1}}\right]$$

Potential without gauge kinetic terms have already been discussed by Hosotani, Oda, Ohnuma & Sakamura



Minimum@ $\pi/2$

A nice feature of this model is that

the radion-Higgs mixing vanishes
at the minimum of Higgs VEV $\theta_H = \pi/2$

$$\partial_{kL}\partial_{\theta_H} V = \frac{k^4 e^{-4kL}}{16\pi^2}\int_0^\infty dw w^3 \left[\partial_{kL}\partial_{\theta_H} v_{eff} - 4\partial_{\theta_H} v_{eff}\right]$$

$$\propto \cos\theta_H = 0 \ @\ \theta_H = \pi/2$$

This greatly simplifies the analysis
of radius stabilization

θ_H independent potential

$$v_{eff}(w;kL) = 24\ln\left[1-\frac{I_0(we^{-kL})K_0^{\kappa_c}(w)}{K_0(we^{-kL})I_0^{\kappa_c}(w)}\right] + 9\ln\left[1-\frac{I_0(we^{-kL})K_0^{\kappa_w}(w)}{K_0(we^{-kL})I_0^{\kappa_w}(w)}\right]$$

$$+3\ln\left[1-c_\phi^2\frac{I_0(we^{-kL})K_0^{\kappa_x}(w)}{K_0(we^{-kL})I_0^{\kappa_x}(w)} - s_\phi^2\frac{I_0(we^{-kL})K_0^{\kappa_w}(w)}{K_0(we^{-kL})I_0^{\kappa_w}(w)}\right]$$

$$-24\ln\left[1-\frac{I_{c_t-1/2}(we^{-kL})K_{c_t-1/2}(w)}{K_{c_t-1/2}(we^{-kL})I_{c_t-1/2}(w)}\right]$$

All terms are written by $\ln\left[1-\frac{I_\beta(we^{-kL})K_0^{\kappa}(w)}{K_\beta(we^{-kL})I_0^{\kappa}(w)}\right]$

$$\frac{I_\beta\left(we^{-kL}\right)}{K_\beta\left(we^{-kL}\right)} \approx \frac{2\Gamma(1-\beta)\sin(\pi\beta)}{\pi\Gamma(1+\beta)}\left(\frac{we^{-kL}}{2}\right)^{2\beta}\left[1+\mathcal{O}\left(\left(\frac{we^{-kL}}{2}\right)^{2\beta}\right)\right]$$

⇒ $\beta \simeq 0$ case dominates

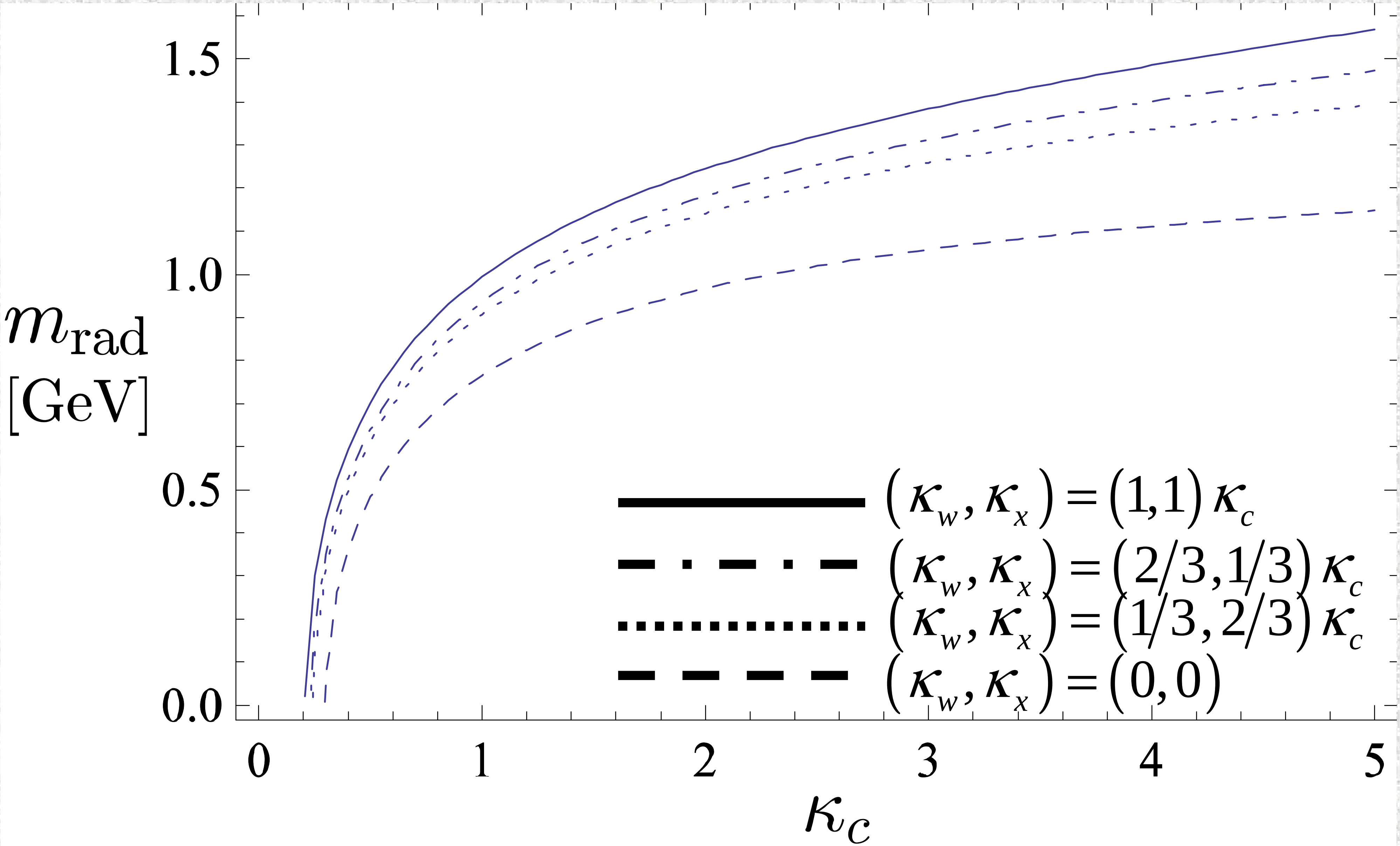
Garriga & Pomarol (2003)

Gauge fields ($\beta = 0$)

Top & Bottom ($\beta = c_t - \frac{1}{2} \sim -0.03$)

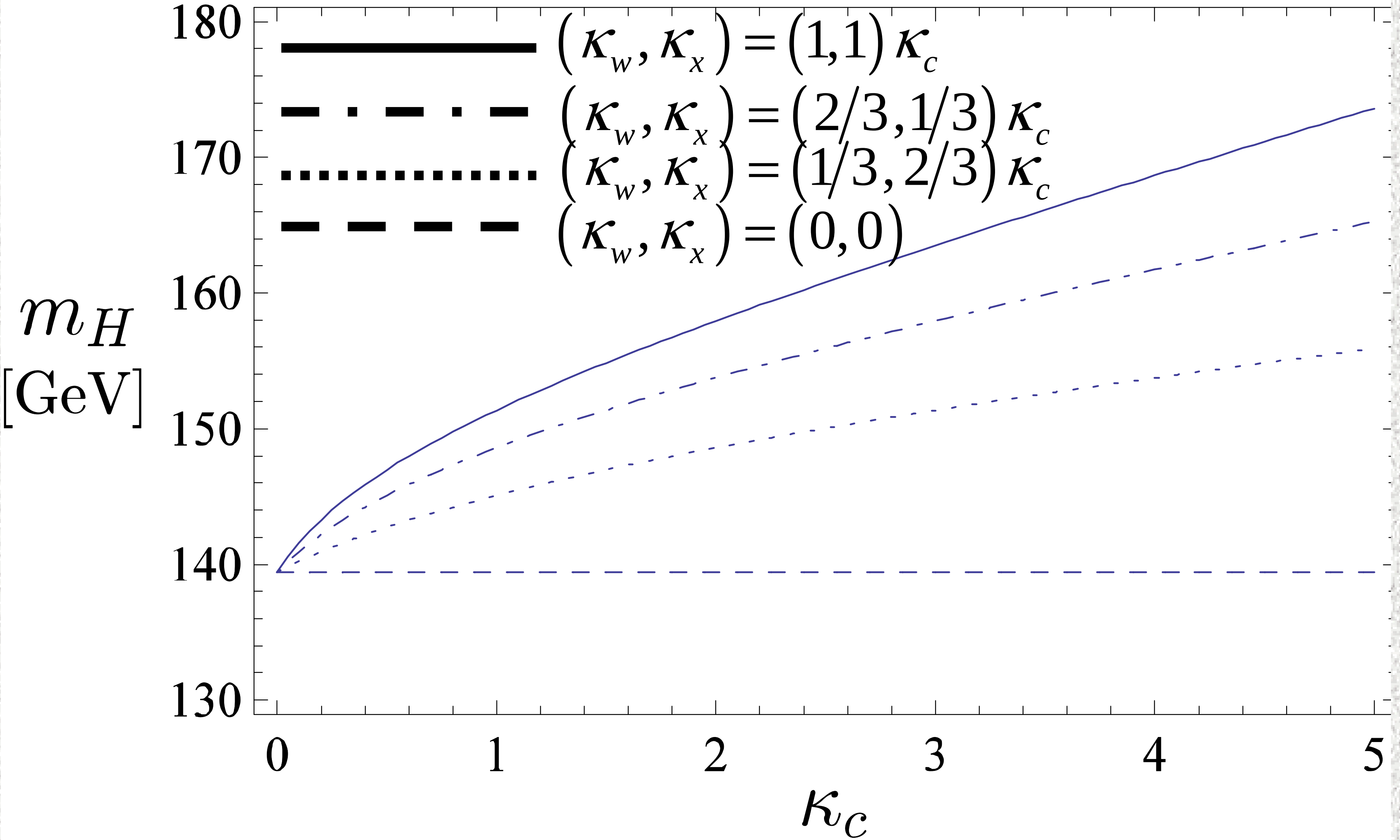
c.f. Graviton ($\beta = 1$)

Radion mass



For the stabilization to work κ_c > 0.2-0.3

Higgs mass

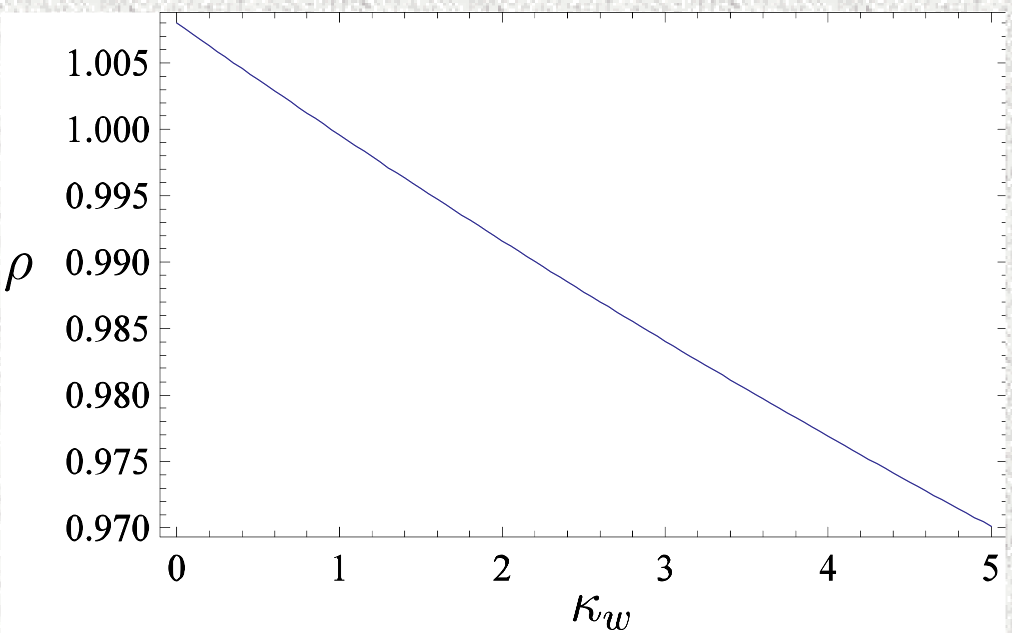


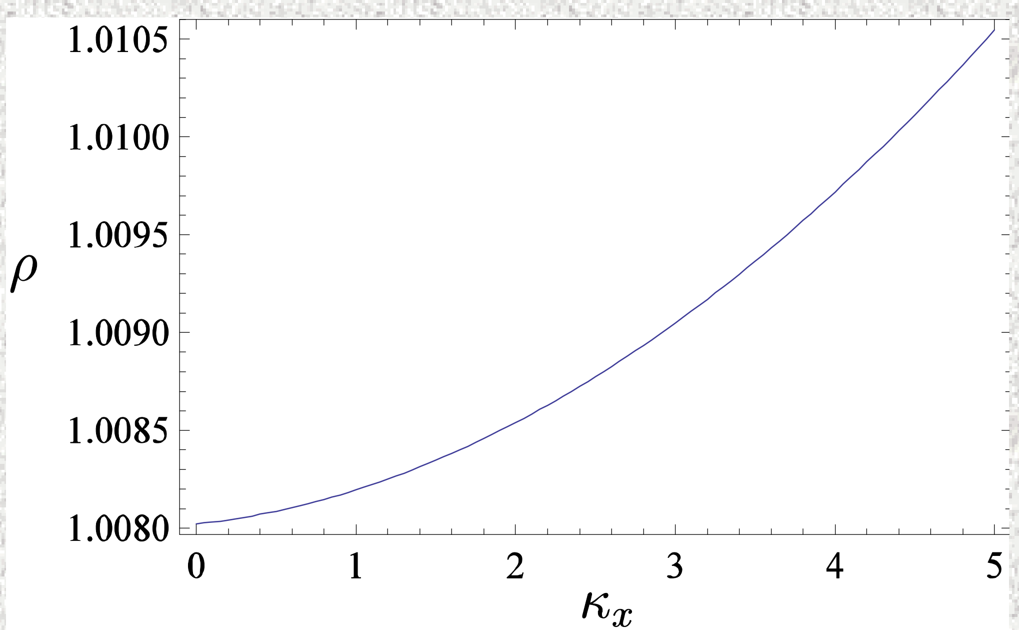
$(\kappa_w, \kappa_x) = (1,1)\,\kappa_c$
$(\kappa_w, \kappa_x) = (2/3,1/3)\,\kappa_c$
$(\kappa_w, \kappa_x) = (1/3,2/3)\,\kappa_c$
$(\kappa_w, \kappa_x) = (0,0)$
m_H [GeV]
180
170
160
150
140
130
0
1
2
3
4
5
κ_c

ρ parameter vs $_{w,x}$

Large $\kappa_{w,x}$ → mode functions of W,Z are repelled away from IR brane where the custodial symmetry exists

→ deviation of ρ parameter





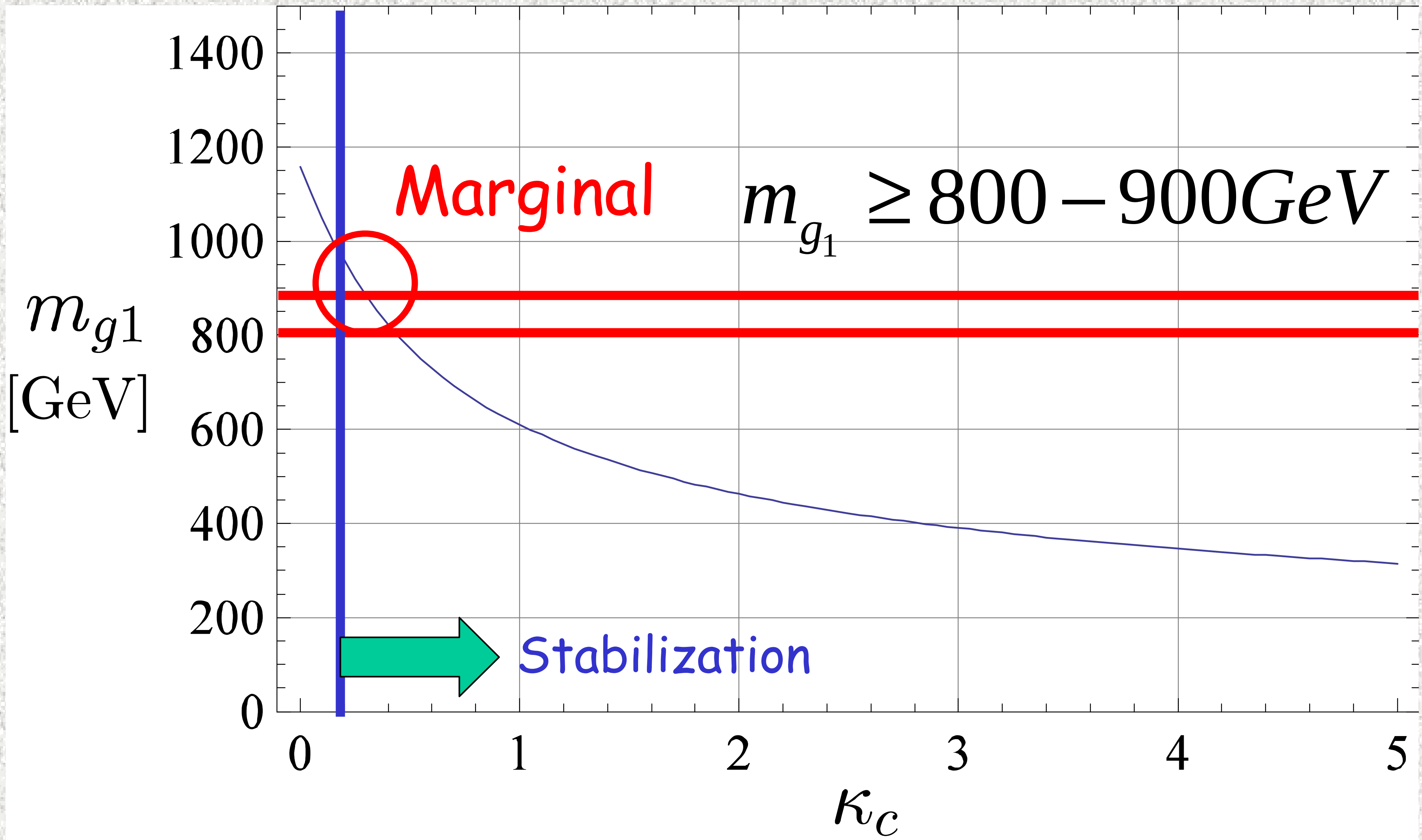
$$1.00989 \le \rho^{\exp} \le 1.01026$$

→

$\kappa_w \sim 1$

$\kappa_x \sim 4$

1st KK gluon mass vs κ_c



Summary

1: Radius stabilization by Casimir energy is economical in GHU

2: IR brane kinetic terms of gauge fields are necessary for the stabilization

3: Magnitude of IR kinetic terms are constrained by ρ parameter and 1st KK gluon mass

4: $M_{radion} \sim 1$ GeV, $M_{Higgs} \geq 140$ GeV

$$m_{rad}^2 = \frac{e^{2kL} - 1}{3kM_5^3} k^2 \partial_{kL} V \simeq \frac{k^5 e^{-2kL}}{48\pi^2 M_5^3} \int_0^\infty dw w^3 \left[\partial_{kL}^2 v_{eff} - 4\partial_{kL} v_{eff} \right]$$

$$m_H^2 = g_A \frac{e^{2kL} - 1}{4k} \partial_{\theta_H}^2 V \simeq \frac{g_A^2 k^3 e^{-2kL}}{64\pi^2} \int_0^\infty dw w^3 \partial_{\theta_H}^2 v_{eff} \, , \; g_4 \equiv \frac{g_A \sqrt{k}}{\sqrt{kL + \kappa_w}}$$

$$\hat{F}^{\kappa}_{\alpha,\beta}(w) \equiv I_\alpha(w) K^{\kappa}_{\beta}\left(we^{kL}\right) - e^{-i(\alpha-\beta)\pi} K_\alpha(w) I^{\kappa}_{\beta}\left(we^{kL}\right)$$

$$I^{\kappa}_{\beta}(u) \equiv I_\beta(u) + \kappa u I_{\beta+1}(u),\ K^{\kappa}_{\beta}(u) \equiv K_\beta(u) - \kappa u K_{\beta+1}(u)$$

$$\begin{pmatrix} A'^{3R}_M \\ A'^{Y}_M \end{pmatrix} = \begin{pmatrix} c_\phi & -s_\phi \\ s_\phi & c_\phi \end{pmatrix} \begin{pmatrix} A^{3R}_M \\ A^{Y}_M \end{pmatrix},\ c_\phi \equiv \frac{g_A}{\sqrt{g_A^2 + g_B^2}},\ s_\phi \equiv \frac{g_B}{\sqrt{g_A^2 + g_B^2}}$$

$$\tau_{UV} \equiv \sum_I (-)^{2\eta_I}\ N_I \int_0^\infty dw w^{D-1} \ln \frac{\mathcal{K}_I(w)}{f_I^{UV}(w)},\quad \tau_{IR} \equiv \sum_I (-)^{2\eta_I}\ N_I \int_0^\infty dw w^{D-1} \ln \frac{\mathcal{I}_I(w)}{f_I^{IR}(w)}$$